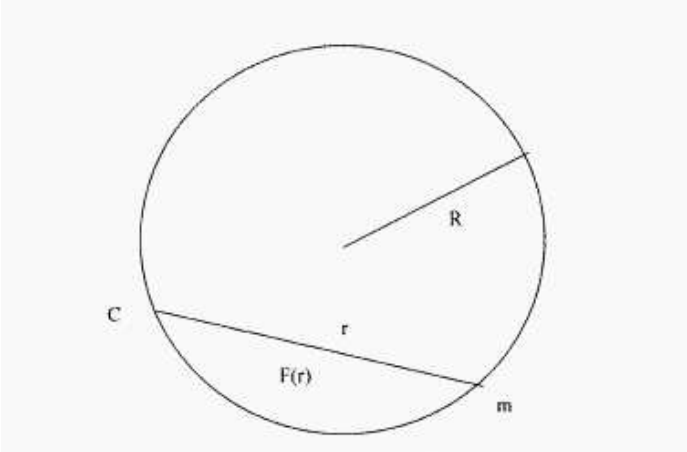
Classical Mechanics

September 12, 2003

Work 2 of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

a) A particle of mass m moves in a circular orbit of radius R under the central force $F(r)$. The center of force C lies at a point on the circle (see Fig). What is the force law?



b) Find the central force which results in the following orbit for a particle:

$$r = a(1 + \cos\theta)$$

where r and θ are the circular coordinates of the particle.

Problem 2

You are asked to drill a hole perpendicular to the face of a homogeneous parallelepiped (dimensions: L length of the long axis, a, b dimensions of the base) such that the period of rotation is shortest. Your answers should address the following topics:

1) Calculate the moment of inertia of the rectangular parallelepiped with respect to its center of mass for a rotation axis perpendicular to one of its faces.

2) Write down and prove the parallel axis theorem.

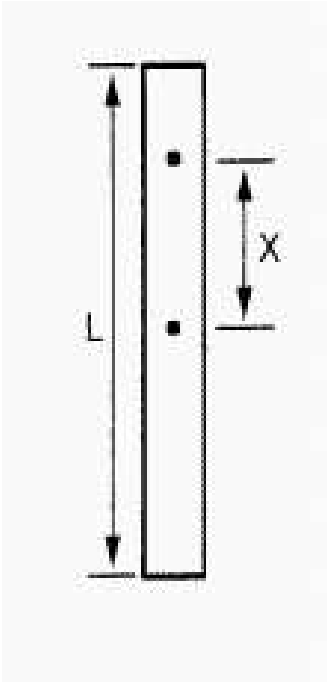
3) Write down the Lagrangian, the Hamiltonian, and the equation of motion for the physical pendulum.

4) What is the general solution of the equation of motion for small angles?

5) What is the period of the pendulum for small oscillations?

6) How far from the center of mass would you drill the hole to minimize the period? (Assume that the mass of the parallelepiped is concentrated towards its long axis and that $L >>$ the lateral dimensions a and b.)

7) For large amplitudes would you expect the period to increase or decrease? Explain.



Problem 3

A particle of mass m_1 with momentum $\mathbf{p}_{1,i}$ undergoes an elastic collision with a particle of mass m_2 and momentum $\mathbf{p}_{2,i}$ traveling in the opposite direction. If particle one leaves the collision at an angle θ_1 with respect to its original trajectory, express the final momentum of the first particle, $\mathbf{p}_{1,f}$, in terms of the initial momentum of the two particles (i.e. $\mathbf{p}_{1,i}$ and $\mathbf{p}_{2,i}$)