Electromagnetism

September 10, 2003

Work 4 of the 5 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

Consider a circular wire loop of radius R in a nonuniform, cylindrically symmetric magnetic field,

$$B = B_0 e^{-\lambda r^2},$$

where B_0 and λ are constants and r is the distance from the axis of symmetry. The loop is located perpendicular to the magnetic field and its center is located at $r = 0$. The loop expands linearly with time: $R = at$, where a is a constant. Find the time when the induced emf in the loop reaches its maximum value.

Problem 2

Consider a particle of mass m, charge q, dropped from rest from height h above the Earth's surface, through a uniform magnetic field of magnitude B oriented horizontally to the Earth's surface (e.g., near the equator where the Earth's magnetic field closely approximates such a field). Assuming $h << R$ (the Earth's radius) and neglecting atmospheric drag:

a) Write down the equations of motion for the particle in terms of the cyclotron frequency $\omega = qB/m$.

b) Solve these equations for the velocity and position of the particle as a function of time.

c) Show that, if ω exceeds a certain magnitude, the particle does not hit the ground. What is the particle trajectory in this case?

Problem 3

The imaginary part of the dielectric function for a semiconductor material can be roughly approximated as

$$\begin{aligned} \text{Im}\ \epsilon(\omega) = &\quad a \quad \text{for} \quad \omega_1 < \omega < \omega_2 \\ \text{Im}\ \epsilon(\omega) = &\quad 0 \quad \text{otherwise.} \end{aligned}$$

Find the real part of the dielectric function and calculate the static dielectric constant of this material.

Problem 4

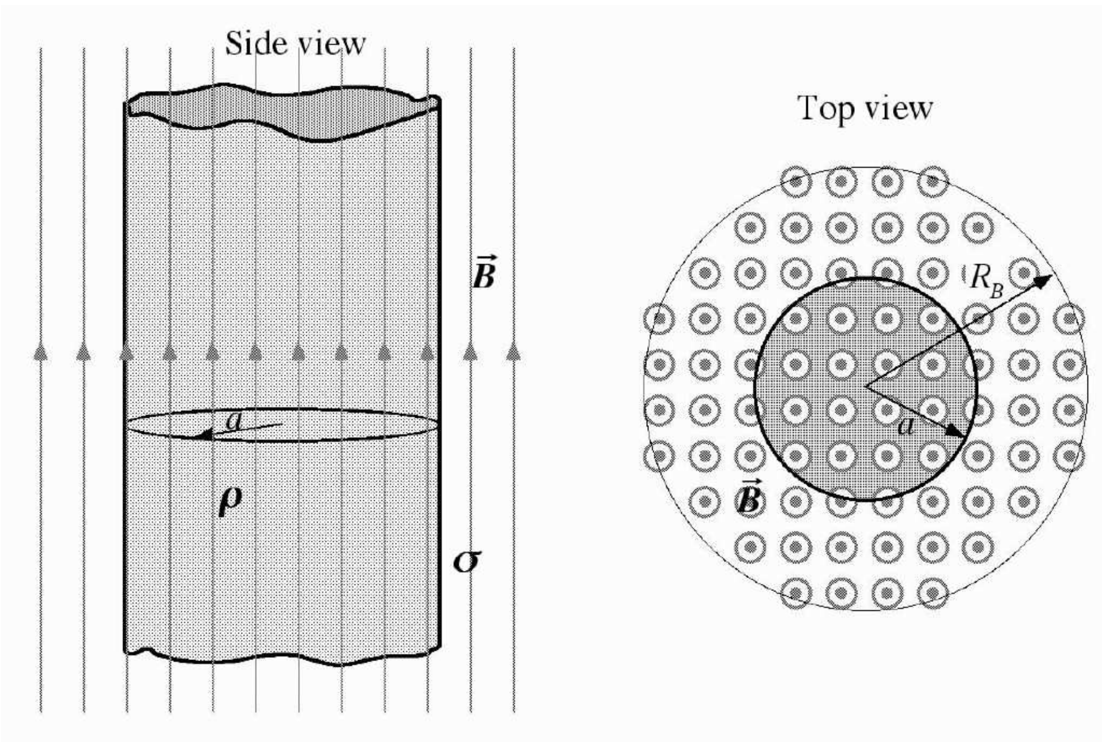
An infinitely long cylinder of insulating material with radius a, permeability $\mu = \mu_0$ and permittivity $\epsilon = \epsilon_0$ has uniform volume charge density $\rho > 0$ and surface charge density σ, and is electrically neutral. It is placed in a constant, uniform magnetic field $\mathbf{B} = B\mathbf{z}$. This magnetic field is cylindrically symmetric with radius R_B (where $R_B > a$) and of infinite extent in the z-direction. The symmetry axes of the cylinder and the magnetic field are collinear. The cylinder is free to rotate about its symmetry axis.

a) Find the electric field $\mathbf{E}$ everywhere.

b) Compute the Poynting vector and momentum density everywhere. Find the angular momentum per unit length of the system, $\hat{L}$, with the cylinder at rest.

c) The magnetic field is now turned off with time dependence $\mathbf{B}(t)$. Determine the electric field induced by the time-varying magnetic field. Assume that any velocity at any point in the charge distribution is small, and any relativistic effects can be neglected.

d) Find the torque per unit length $\hat{\tau}$ on the cylinder as a function of time and determine the mechanical angular momentum per unit length of the cylinder as a function of time. Assume that any velocity at any point in the charge distribution is small, and any relativistic effects can be neglected.



Problem 5

The neutral pion (π^0) is an unstable particle with mass of 135 MeV$/c^2$ that decays (almost always) into two photons, with a lifetime of approximately 8×10^{-17}s. Consider a monochromatic, parallel beam of π^0s with energy of 500 GeV.

1. What fraction of the particles has not yet decayed after a flight path of 0.5 mm from the point where the pions were produced? (2 points)

2. What is the minimum and maximum energy of the decay products (photons) observed in the lab? (4 points)

3. A planar photon detector is located 10 m downstream, facing the beam. What is the minimum and maximum separation between the impact points of the two photons from the decay of a single pion? (4 points)

Note: 1 MeV$= 10^6$eV, 1 GeV$= 10^9$eV.
Hint: For part 3, you can safely assume that all pions decay at a single point, provided you justify this assumption.