Classical Mechanics

February 2, 2005

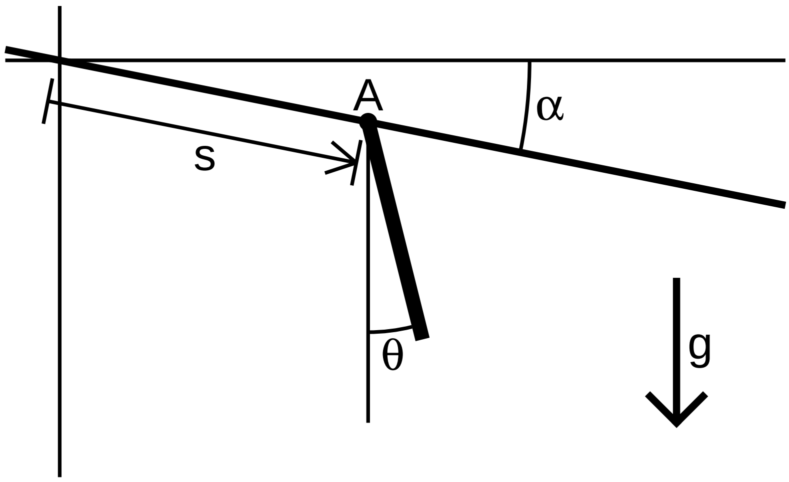
Work 2 of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

(Hearn)
Essay Question: Describe a method for measuring the radius of the Earth that would have been applicable 1000 years ago. You will be graded 2/3 on English writing including clarity of presentation. Only a few paragraphs are needed, but they should be written at a level where a freshman in college can understand them.

Problem 2

(Pate)
A uniform rod of mass m and length $2L$ is constrained to move in a vertical plane. One end of the rod, marked A in the figure, is attached with a sliding frictionless pivot to a track inclined at an angle α with respect to the horizontal axis, where $0 \leq \alpha < \pi/2$.



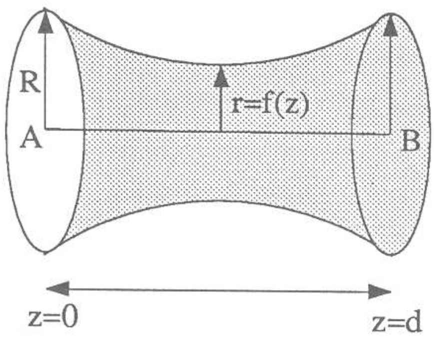
A) Find the equations of motion for the coordinates s and θ.

B) Find the conditions under which a purely translational motion ($\theta =$ constant) is possible.

C) Find the condition under which the motion described in B) is stable.

Problem 3

(Kiefer)
A soap film is stretched between two coaxial circular rings of equal radius R. The distance between the two rings is d (see Figure). You may ignore gravity. Find the shape of the soap film. (Hint: When a film of soap is stretched across a frame, surface tension forces the film to adjust itself so that its surface area is a minimum).



a) Since the problem is axially symmetric about the line AB, the surface can be described by specifying its radius as a function of distance along the symmetry axis (z-direction), $r = f(z)$. Show that the surface area for a given function $f(z)$ can be written as:

$$S = \int_{z=0}^{d} 2\pi f \sqrt{df^2 + dz^2} = \int_{z=0}^{d} 2\pi f \sqrt{(f')^2 + 1}\, dz = \int_{z=0}^{d} G\, dz$$

with

$$G = 2\pi f \sqrt{(f')^2 + 1}\ , \qquad f' = \frac{df}{dz}$$

b) Since the surface area has to be a minimum (extremal) we can use the calculus of variations to solve this problem. What differential equation has to be satisfied by $f(z)$? Give the explicit form of the differential equation in terms of $G(z, f, f')$.

c) At this point we note that the differential equation does not depend explicitly on the z-coordinate. This implies that there exists a first integral of this problem (a conserved quantity), namely $C = G - f'\partial G/\partial f'$. Show that this combination is indeed a constant in z.

d) Use the constraint you found in c) to obtain an explicit expression for $f(z)$. Determine the two constants of integration from the boundary conditions of the problem.

e) One of the two boundary conditions leads to a transcendental equation that cannot be solved analytically. However, for $R/d \ll 1$, no solution exists (do **not** prove). What is the physical interpretation of this finding?