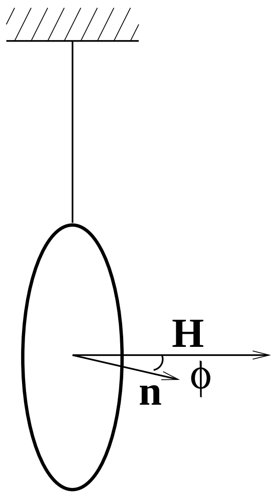
Electromagnetism

January 31, 2005

Work 4 of the 5 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

(Vasiliev)
A circular metal ring of area S, resistance R, and inductivity L is attached to a non-conducting thread and placed in a horizontal uniform periodic magnetic field $\mathbf{H} = \mathbf{H}_0 \cos \omega t$. The angle between $\mathbf{H}$ and the normal to the surface of the ring $\mathbf{n}$ is equal to ϕ. Calculate the average torque acting on the ring. Assume that the ring is sufficiently massive that ϕ can be considered to be constant.



Problem 2

(Curran)
The electric field vector of a time-harmonic, uniform plane wave is given by

$$\vec{E}(z,t) = \vec{e}_x 2\sin(10^8 t + 0.5z) - \vec{e}_y 2\cos(10^8 t + 0.5z)\ (V/m) \tag{1}$$

The wave is propagating in a lossless medium of permeability $\mu_0 = 4\pi \cdot 10^{-7} H/m$. Calculate the following:

a) The direction of propagation of the wave.

b) The phase velocity of the wave.

c) The relative permittivity of the medium.

d) The polarization of the wave (if elliptical or circular, make sure to indicate also whether it is left-hand or right-hand).

Problem 3

(Nakotte)

a) Assume that a conductor with non-zero resistivity ρ is placed in a slowly varying external magnetic field $\mathbf{H}(\mathbf{t})$. At a time $t = t_0$, ρ suddenly drops to zero. Apply Maxwell's equation and Ohm's law to determine what will happen to the magnetic field inside the conductor for $t > t_0$.

b) The experimentally observed phenomenon of superconductivity is not equivalent to $\rho = 0$. Given that the behavior of superconductors in the magnetic field is described by Maxwell's equations plus the London equation

$$\mathbf{H} + \lambda^{\mathbf{2}} \nabla \times \nabla \times \mathbf{H} = \mathbf{0}$$

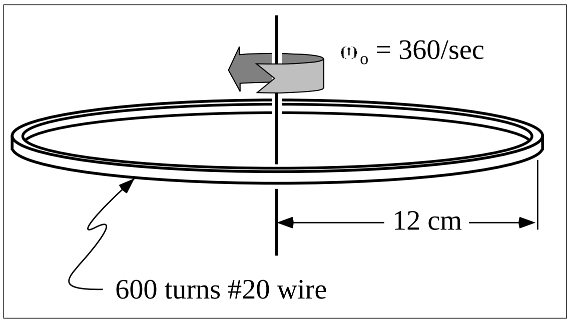
where λ has the dimensionality of length, calculate the behavior of H inside a superconducting plate of thickness d and infinite length placed in a constant external magnetic field H_0 applied in the direction to the surface of the plate (z direction). Make a sketch of your solution and explain the significance of λ.

Problem 4

(Kanim)
One simple way to model current in conductors is to assume a resistive force on the charge carriers that is linearly proportional to the velocity of the charge carriers *relative to* the velocity of the metal: $f_{res} = -\beta u$. Here $\beta = eC\rho$ where e is the charge of the electron, C is the number of valence electrons per unit volume and ρ is the resistivity.

(a) [2 points] From the values given below, find the value of C for copper.

In 1916 R.C. Tolman and T.D. Stewart devised an experiment to determine whether the charge carriers in metals were electrons. They reasoned that if the charge carriers in a metal have mass, they also have momentum. A 12-cm radius coil of 600 turns of insulated copper wire forms the basis of this simplified version of their experiment. The conductor of the #20 wire has a radius of 0.4mm. The coil is rotated (counterclockwise as seen from above) about its axis with an angular velocity of 360 radians per second and then brought suddenly to rest.



(b) [5 points] A ballistic galvanometer was used to measure the total charge that passed from the coil to an external circuit while the coil was brought to rest. We can model this external circuit as a resistor and an inductor in series with the coil. The total external resistance $R_{ext} = 25\Omega$. For one run a total charge of 2.85×10^{-9} Coulombs passed through the external circuit. Find the ratio of the charge carrier's charge-to-mass implied by this measurement.

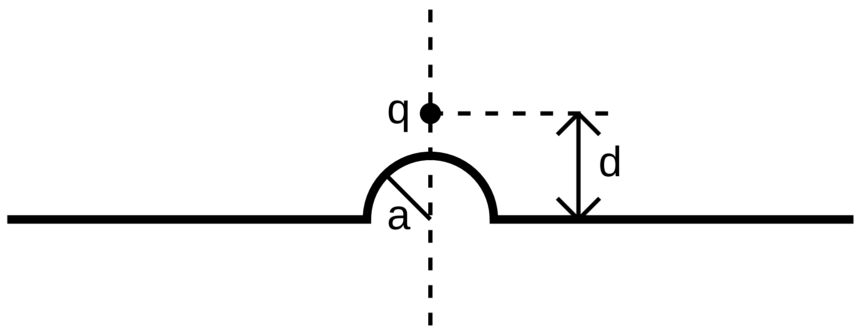
(c) [3 points] One obstacle to this measurement was the effect of small variations in the strength of the earth's magnetic field with time. Explain (i) why this variation would affect the voltage measurement; and (ii) how a second (identical) stationary coil could be used to effectively eliminate this problem.

Mass of an electron $9.1 \times 10^{-31}\, kg$
Charge of an electron $1.6 \times 10^{-19}\, Coul$
Permeability of free space $4\pi \times 10^{-7}\, N/A^2$
Permittivity of free space $8.85 \times 10^{-12}\, N \cdot m^2/C^2$

Copper $Z = 29$ $A = 63$:
1 valence electron per atom
Resistivity $\rho = 1.7 \times 10^{-8}\, \Omega \cdot m$
Mass Density $D = 9 \times 10^3\, kg/m^3$

Problem 5

(Armstrong)
Consider a grounded conducting plane with a hemispherical boss of radius $a < d$ on it. A point charge q is located a distance d from the plane on the axis of the boss which is perpendicular to the plane, as shown in the figure:



(a) Compute the potential above the conductor.

(b) Compute the charge density on the conductor, both on the planar part and on the boss.

Hint: Use image charges (three in addition to the given charge)