Quantum Mechanics

January 28, 2005

Work 4 of the 5 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

(Kyle)
A particle in an infinitely deep one-dimensional rectangular potential well

$$V = \infty; \quad x < 0 \ \text{ or } \ x > a$$

$$V = 0; \quad 0 < x < a$$

is in a state described by the Schrödinger wave function

$$\psi(x) = Ax(a - x)$$

where A is a constant.
Find the probability distribution for the different energy eigenstates of the particle and calculate the average value and the dispersion of the energy using that probability distribution. You might find the following well-known sums useful,

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \quad , \qquad \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$$

Problem 2

(Curran)
Define the work function of a metal and show pictorially how this relates to the Fermi level and electron emission. What is the Schottky effect? Draw a band structure diagram demonstrating the flow of electrons and holes between a low workfunction metal and a p-type semiconductor.

Problem 3

(Burkardt)
An electron is in a state described by the wave function

$$\psi(r,\theta,\phi) = \mathcal{N} r e^{-ar} \sin(\theta)\sin(\phi).$$

where $\mathcal{N}$ is a normalization factor such that the wave function is normalized to 1.

a. Suppose L_z is measured in this state. What are the possible outcomes for the measurement of L_z (list only those with nonzero probability) and what are the respective probabilities? (2 points)

b. What are the expectation value and standard deviation for a measurement of L_z in this state? (2 points)

c. Suppose some value of L_z has been measured in part a.) (pick one). Write down the wave function describing the state after the measurement. (2 points)

d. Suppose now that $\vec{L}^2$ is measured in the original state (the one described by the wave function above). What are the possible outcomes for the measurement of $\vec{L}^2$ (list only those with nonzero probability) and what are the respective probabilities? (2 points)

e. Suppose some value of $\vec{L}^2$has been measured in part b.) (pick one). Write down the wave function describing the state after the measurement. (2 points)

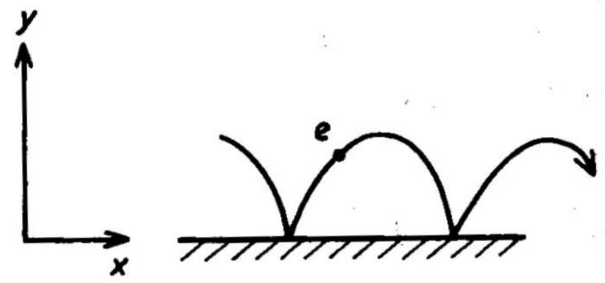
$$Y_0^0 = \frac{1}{\sqrt{4\pi}} \qquad Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}}\sin(\theta)e^{\pm i\phi}$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos(\theta) \qquad Y_2^{\pm 2} = \sqrt{\frac{15}{32\pi}}\sin^2(\theta)e^{\pm 2i\phi}$$

$$Y_2^{\pm 1} = \mp\sqrt{\frac{15}{8\pi}}\sin(\theta)\cos(\theta)e^{\pm i\phi} \qquad Y_2^0 = \sqrt{\frac{5}{16\pi}}\left(3\cos^2(\theta) - 1\right)$$

Problem 4

(Kiefer)
An electron moves above an impenetrable conducting surface. It is attracted toward this surface by its own image charge so that classically it bounces along the surface as shown in the Figure.



a) Write the Schroedinger equation for the energy eigenstates and energy eigenvalues of the electron (call y the distance above the surface). Ignore inertial effects of the image.

b) What is the x and z dependence of the eigenstates? (Hint: Separation of variables)

c) What are the remaining boundary conditions?

d) Find the ground state and its energy. (Hint: They are closely related to those for the usual hydrogen atom, including a coordinate transformation).

e) What is the **complete** set of discrete and/or continuous energy eigenvalues?

f) For the state of lowest energy, find the average distance of the electron above the conductor's surface.

Useful formulas:

Radial Schroedinger equation for hydrogen-like atoms:

$$-\frac{\hbar^2}{2mr^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) - \frac{Ze^2}{r}R + \frac{l(l+1)\hbar^2}{2mr^2}R = ER$$

where $r, R(r), E, Z, m$ are the radial coordinate, the radial wave function, the energy eigenvalue, the nuclear charge, the electron mass, and $\hbar = h/2\pi$.

The ground state wave-function of the hydrogen-like atom is: $(l = 0)$

$$R_{10}(r) = 2\left(\frac{Z}{a_B}\right)^{3/2} \exp\left(-\frac{Zr}{a_B}\right)$$

where $a_B = \hbar^2/(me^2)$ is Bohr's radius.

Problem 5

(Papavassiliou)
A particle of mass m moving in a one-dimensional potential $U(x)$ is described by the following wavefunction, independent of time:

$$\psi(x) = \begin{cases} A\left[1 - \frac{1}{2}\left(\frac{x}{x_0}\right)^2\right], & |x| \leq x_0 \\ Be^{-\alpha|x|}, & |x| > x_0 \end{cases}$$

where α is real ($\alpha > 0$). Write the potential as a function of x for $-\infty < x < \infty$ in terms of the parameters m and x_0.