Classical Mechanics

September 14, 2005

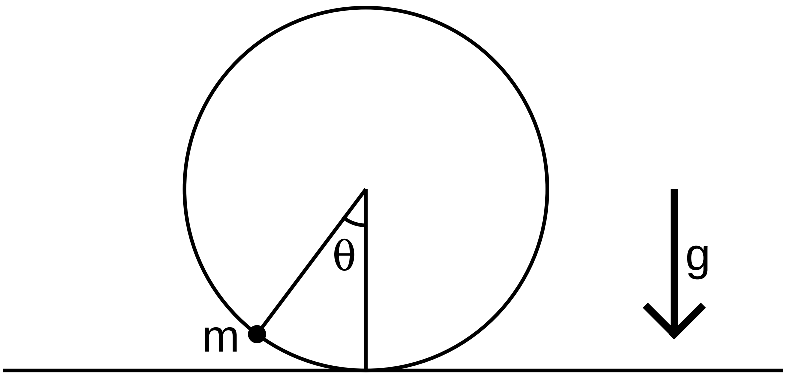
Work 2 of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

(Curran)
A particle of mass m moves in a 1-dimensional potential $U(x) = A|x|^n$, where A and n are positive constants. Give the dependence of the period τ on the energy E up to a constant factor independent of E.

Problem 2

(Pate)
A hollow cylinder of mass m and radius r has a point mass m attached to a point on the circumference. It rolls without slipping on a rough horizontal plane. We describe the system by the angle θ as shown in the figure.



a.) Obtain the equation of motion by the Lagrangean method.

b.) Find the frequency of small oscillations.

Problem 3

(Kiefer)
Poisson brackets provide some of the most fundamental insights into classical mechanics. They are useful in describing conserved quantities and also provide the classical analog to the Heisenberg picture in quantum mechanics. The present problem is concerned with the former aspect.

a.) [4 points] Prove the following relationship: If f and g are constants of motion, then the Poisson bracket $[f, g]$ is also a constant of motion. In other words, show: $f, g = constant \Rightarrow [f, g] = constant$.

b.) [4 points] Show that L_z, the z-component of the angular momentum, is a constant of motion if L_x and L_y are constants of motion.

c.) [2 points] Justify the following true statement: In a planetary system in which all planets are moving in the $z = 0$ plane, the z-component of the angular momentum of each planet is a constant of motion regardless of the interaction between the planets. Why do the z-components of the individual angular momenta of the Earth and the Moon nevertheless change continuously in reality?

Some possibly useful relationships **(do not prove)**:

Jacobi identity for Poisson brackets:

$$[f, [g, h]] + [g, [h, f]] + [h, [f, g]] = 0$$

The completely antisymmetric tensor of rank three is defined as

$$\epsilon_{ikl} = \begin{cases} 1 & \text{for } ikl \text{ an even permutation of } 123 \\ -1 & \text{for } ikl \text{ an odd permutation of } 123 \\ 0 & \text{else} \end{cases}$$