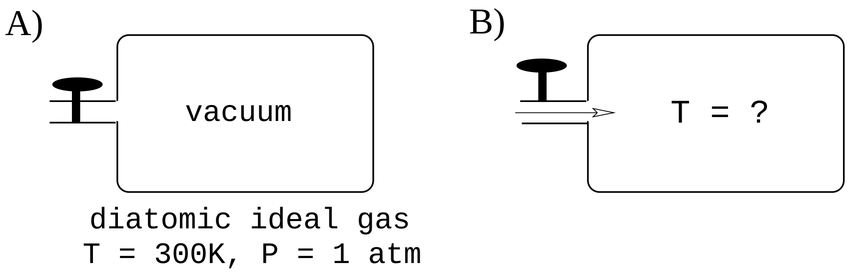
Statistical Mechanics

September 14, 2005

Work 2 of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

(Vasiliev)
A vacuum chamber is surrounded by a *diatomic* ideal gas at $P = 1$ atm and $T = 300$ K. Initially, the air is pumped out of the chamber and the valve is closed (Fig. A). Then, the valve is instantly opened and the gas fills the chamber (Fig. B). Find the temperature of the gas inside the chamber. Assume that the walls of the chamber are absolutely rigid, that the walls and the valve do not absorb heat or allow it to escape and that the pressure outside is constant.



Problem 2

(Urquidi)

Consider an Ideal Gas molecule AB which undergoes the dissociation reaction

$$AB \longrightarrow A + B$$

If n_A, n_B, and n_{AB} are the concentrations (the number of molecules per unit volume) of each molecule respectively, derive the law of mass action:

$$\frac{n_{AB}}{n_A n_B} \equiv K(T) = \frac{V f_{AB}}{f_A f_B} \exp\left(\frac{w_0}{kT}\right) = \left[\frac{(m_A + m_B)h^2}{2\pi m_A m_B kT}\right]^{3/2} \frac{j^0_{AB}}{j^0_A j^0_B} \exp\left(\frac{w_0}{kT}\right)$$

where f_A, etc. are the partition functions per molecule, V is the volume of the container, and j^0_A, etc. are the partition functions for the internal degrees of freedom of each molecule. The zero of energy for each molecule is chosen at the ground state (not including the zero point energy of the vibration) of the respective molecule so that $w_0 = \epsilon^0_A + \epsilon^0_B - \epsilon^0_{AB}$ is the difference in the energy zeros.

Problem 3

(Engelhardt)

Many one-dimensional models can be solved by the method of the transfer matrix if the interaction is limited to nearest neighbors. Consider the Hamiltonian

$$H = \sum_{i=1}^{N} K(s_i, s_{i+1})$$

with $s_{N+1} \equiv s_1$. K can be any function of the discrete variables s_i and s_{i+1}.

a.) Prove that the partition function satisfies

$$Z \equiv \sum_{s_1, s_2, \ldots, s_N} \exp(-\beta H) = \mathrm{Tr}\,(Q^N)$$

where the transfer matrix Q is defined as

$$Q_{ab} \equiv \exp(-\beta K(a, b)).$$

b.) In the limit of large N, why can one well approximate

$$\ln Z = N \ln \lambda$$

where λ is the largest eigenvalue of Q?

c.) Use this method to solve the one-dimensional Ising model, which is described by the following Hamiltonian, where the spin variables s_i can take the two values ± 1,

$$H = -J \sum_{i=1}^{N} s_i s_{i+1}.$$

Start by showing that

$$Q = \begin{pmatrix} \exp(\beta J) & \exp(-\beta J) \\ \exp(-\beta J) & \exp(\beta J) \end{pmatrix}.$$

Give $\ln Z$ in the large N limit and, from this, derive $E(T) - E(T = 0)$, i.e., the mean energy of the system at temperature T relative to the zero-temperature value.