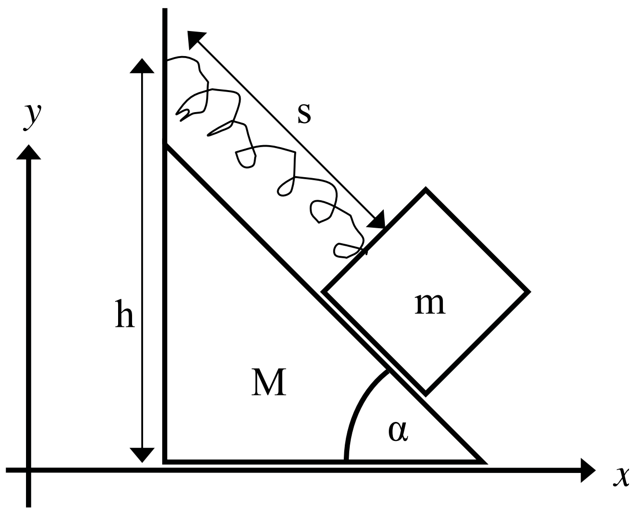
Classical Mechanics

September 27, 2006

Work 2 of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

A block of mass m is attached to a wedge of mass M by a spring with spring constant k. The inclined frictionless surface of the wedge makes an angle α with the horizontal. The wedge is free to slide on a horizontal frictionless surface, as shown in the figure:



Solve the problem in the coordinate system shown in the figure.

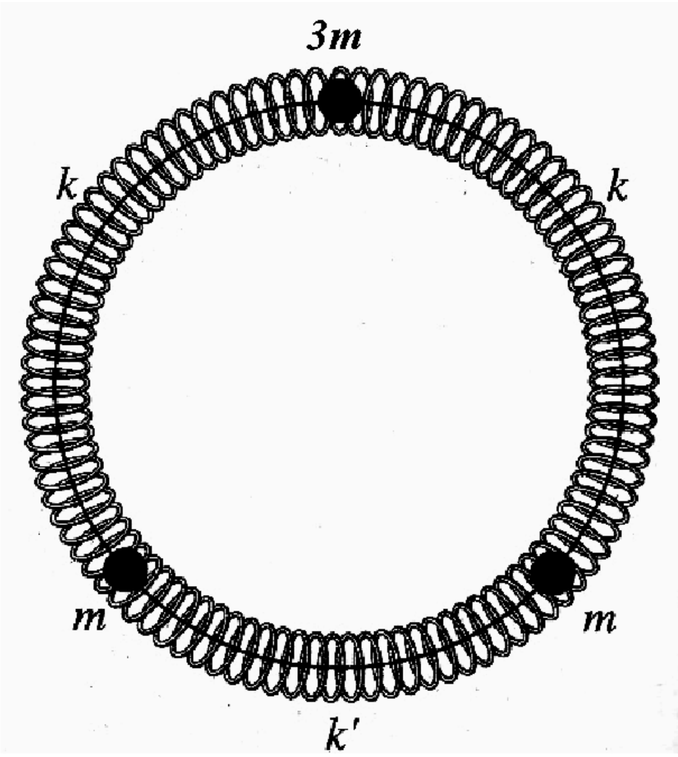
(A) Given that the relaxed length of the spring alone is d, find the value s_0 when both the block and the wedge are at rest.

(B) Find the Lagrangean for the system as a function of the x-coordinate of the wedge and the length of the spring s. Write down the equations of motion for x and s.

(C) How many natural frequencies does the system have and what are they?

Problem 2

Three beads of masses m, m and $3m$ are constrained to slide along a frictionless, circular hoop of radius R. Each mass is connected to each other mass by springs of length a and force constants k and k' (see figure for equilibrium positions which are located at $120°$ angular separations). The largest mass is initially displaced $10°$ clockwise from its equilibrium position and the other two smaller masses are held in place. The three masses are then simultaneously released from rest.

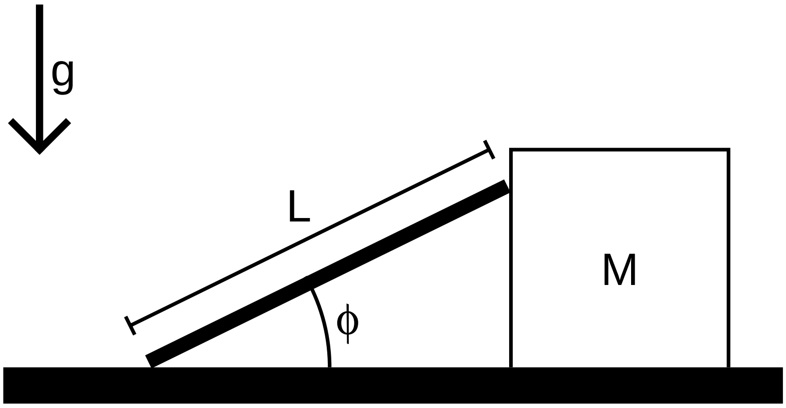


(A) Find the normal frequencies and the normal modes of oscillation.

(B) Solve for the resulting motion of each mass.

Problem 3

A rigid rod of length L and mass m is placed with one end on a smooth table and the other leaning against a smooth block of mass M. All motion is constrained to the plane of the paper, and both the rod and the block are free to slide on the table.



Find the forces on the rod immediately after the rod is placed in its initial position (at $\phi = \phi_0$) and released.