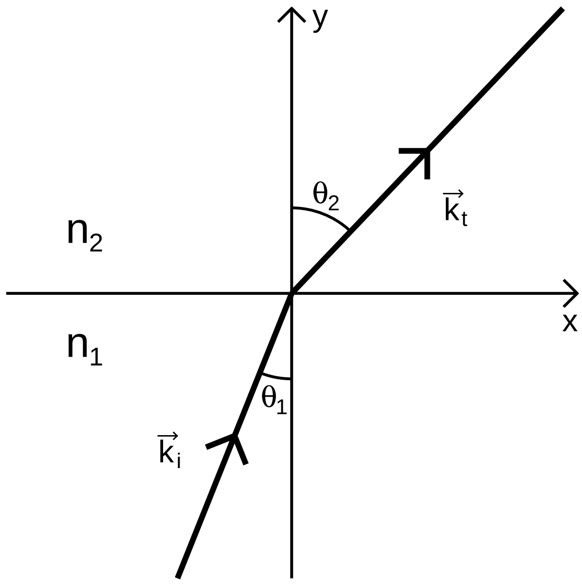
Electromagnetism

September 25, 2006

Work 4 of the 5 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

Consider a plane electromagnetic wave incident on the interface between two dielectric media (characterized by n_1 and n_2) at the angle θ_1 as shown:



The incident and transmitted electric fields are

$$\vec{E}_i(\vec{r}, t) = \vec{E}_{0i} e^{i(\vec{k}_i \cdot \vec{r} - \omega t)} \qquad \vec{E}_t(\vec{r}, t) = \vec{E}_{0t} e^{i(\vec{k}_t \cdot \vec{r} - \omega t)}$$

(A) Show that, for $\theta_1 > \theta_{1c}$ (the critical angle for total internal reflection), the second (n_2) medium becomes absorbing (i.e., the transmitted amplitude decays exponentially in the medium). What is the critical angle θ_{1c} (give a formula)?

(B) For $n_1 = 1.5$ (glass) and $n_2 = 1$ (air), determine the numerical value of θ_{1c} for $\lambda = 600\,nm$.

(C) What is the exponential decay length for this wavelength when $\theta_1 = 1.03\,\theta_{1c}$?

Problem 2

An infinite cylinder with charge density λ and current flow I is at rest in the reference frame S. Find the speed of a reference frame S' where the electric field is zero; i.e., in that frame one observes a pure magnetic field.

Problem 3

When a molecule with no permanent dipole moment is placed in an electric field $\vec{E}$, the nuclei of the atoms that comprise the molecule tend to move in the direction of $\vec{E}$, whilst the electrons, important for chemical bonding, tend to move in the opposite direction. The molecule will settle into an equilibrium, where the center of positive charge is displaced from the center of negative charge by a small amount, thus producing a dipole that was induced by the presence of the electric field. Consider a material made of a non-conducting substance. Let us suppose that the effect of the electric field $\vec{E}$ on this dielectric is to induce an electric dipole moment of $10^{-30}\,Cm$ per molecule.

(A) What is the surface polarization charge density on a surface making an angle of 45° with $\vec{E}$, if the number density is 10^{21} molecules/cm^3?

(B) The resulting induced dipole was produced through a shift in the charge density of the material. What is the effective electron shift, i.e., the number of electrons which would have to be added per molecule at the surface of the material, that would give rise to this surface charge density?

Problem 4

Consider the hemispherical spatial region $I = \{(r, \theta, \phi) : 0 < r < R, 0 \leq \theta < \pi/2, 0 \leq \phi < 2\pi\}$ bounded by the disc $D = \{(r, \theta, \phi) : 0 \leq r < R, \theta = \pi/2, 0 \leq \phi < 2\pi\}$ and the hemispherical shell $S = \{(r, \theta, \phi) : r = R, 0 \leq \theta < \pi/2, 0 \leq \phi < 2\pi\}$, where (r, θ, ϕ) are the usual spherical coordinates and R is fixed. Let I be a vacuum, D a conductor at zero potential, and S a conductor at potential V (the two conductors are separated by a very thin insulating ring). Solve for the electrostatic potential in the region I by using an expansion in Legendre polynomials as the ansatz for the solution. Determine the first three nontrivial terms of the expansion explicitly. The orthogonality relation for Legendre polynomials is

$$\int_{-1}^{1} dx\, P_{l'}(x) P_l(x) = \frac{2}{2l+1}\, \delta_{l'l} \quad \text{and} \quad \mathrm{P}_l(1) = 1.$$

Problem 5

A charged particle is placed in constant uniform mutually orthogonal electric and magnetic fields

$$\mathbf{E} = aE_0\hat{\mathbf{y}} \quad ; \quad \mathbf{B} = bE_0\hat{\mathbf{z}},$$

where $a < b$. The initial velocity of the particle is zero in the laboratory frame X.

a) Find the drift velocity of the particle in the laboratory frame X. What is the direction of the particle drift?

b) What happens if $a > b$?

c) In cases (a) and (b), sketch the particle trajectory in the $x - y$ plane in the laboratory frame X.