Classical Mechanics

February 7, 2007

Work 2 of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

If the solar system were immersed in a homogeneous cloud of weakly interacting massive particles (WIMPs), then objects in the solar system would experience gravitational forces from both the sun and the cloud of WIMPs such that

$$F_r = -\frac{k}{r^2} - b \cdot r \ .$$

Assume that the force due to the WIMPs is very small, namely that $b \ll k/r^3$ for all r of interest.

(A) Give an argument that shows that the motion of the objects can be considered as 2-dimensional.

(B) Write down the Lagrangian in planar polar coordinates for this system.

(C) Using this Lagrangian, confirm that the angular momentum is conserved.

(D) Find the frequency of the radial oscillations $\omega_R = 2\pi/T_R$ for a nearly circular orbit of radius r_0.

(E) Show that the perihelion precesses backwards by an angle $\delta\theta$. That is, it takes longer to complete one complete orbit than it does to complete one radial oscillation. Determine $\delta\theta$.

(F) Show that the rate of precession of the perihelion of this orbit to lowest order is given by:

$$\alpha = \frac{\delta\theta}{T_R} = \frac{3}{2} b \sqrt{\frac{r_0^3}{m \cdot k}} \ .$$

Some hints:

1. $\sqrt{1 \pm x} \approx 1 \pm x/2$
2. $1/\sqrt{1 \pm x} \approx 1 \mp x/2$

Problem 2

A $100\,N$ weight is hanging from one end of a rope. The rope winds around a circular ceiling beam and its other end is held by Jack, who holds the rope statically in place. If the rope wraps a total of 270 degrees around the beam and the coefficient of static friction between rope and beam is $\mu = 0.5$, what is the minimum tension with which Jack must hold the rope?

Problem 3

A double pendulum consists of two simple pendula, with one pendulum suspended from the bob of the other, cf. figure below. The two pendula have equal lengths l, have bobs of equal mass m and both pendula are confined to move in the same plane. Additionally, the top of the double pendulum is allowed to move freely along a line in the horizontal direction. Find the Lagrange equations of motion for the system. Do not assume small angles.