Classical Mechanics

September 24, 2008

Work 2 of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

Consider an ideal spherical pendulum, i.e., a point-like bob of mass m attached to a rod of negligible mass and length R which can pivot freely around the origin. Thus, the bob is constrained to move on a sphere of radius R around the origin; it is subject to gravity.

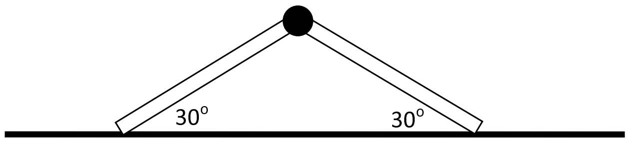
a. Give the Lagrangean and the equations of motion using the polar and azimuthal angles of the spherical coordinate system as degrees of freedom. Eliminate the cyclical coordinate in favor of the corresponding conserved angular momentum L_z.

b. Find solutions associated with circular orbits at constant height, and give their angular momentum L_z and period.

c. Consider small perturbations around such a solution, where the perturbed solution retains the same L_z. Give the equation of motion for the perturbation to leading order. Find the period associated with the perturbation and compare to the period of the underlying circular motion.

Problem 2

Two identical thin beams of mass m and length l are connected by a frictionless hinge. The system rests on a smooth surface as shown in the figure below, with the hinge held at the position shown by a thread. At $t = 0$, the thread is cut. In your analysis of the problem, treat the beams as extended objects of constant density. However, to simplify the analysis, you may neglect the masses of the hinge and the thread.



a. Calculate the moment of inertia of each of the beams for rotations about its center of mass. *(2 points)*

b. What is the (linear) speed of the hinge as it hits the floor? *(7 points)*

c. Find the time it takes for the hinge to hit the floor, expressing this in terms of an integral that you do not need to solve explicitly. *(1 point)*

Problem 3

Find the solution to an overdamped harmonic oscillator started with an initial amplitude of A and initial speed of zero. A overdamped harmonic oscillator consists of a spring and dashpot in parallel with the damping coefficient high enough such that no oscillation occurs.