Classical Mechanics

September 2, 2009

Work 2 of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

A particle of mass m is constrained to move on the paraboloid of revolution $z = c\rho^2$ in the potential

$$V = a\rho^2 + b\rho^4$$

where $c > 0$ and ρ is the distance of the particle from the z axis (*i.e.* $\rho^2 = x^2 + y^2$).
(a) Reduce the solution of the problem to an integration, using the Lagrangian method.
(b) Let $a, b > 0$. Derive the condition for a circular orbit about the z axis. Obtain *all* independent coordinates as a function of time for motions *near* the circular orbit.

Problem 2

Consider, in one dimension, the motion of a particle of mass m under the influence of two springs; each of the springs has one end fixed, and the other attached to the particle. The fixed ends of the two springs are a distance a apart. The springs obey Hooke's law, have zero un-stretched lengths and force constants k_1 and k_2, respectively.

a. Using the displacement q of the particle from the fixed end of one of the springs as the generalized coordinate, find the Lagrangian and the corresponding Hamiltonian. Is the energy conserved? Is the Hamiltonian conserved?

b. Introduce a new coordinate Q defined by

$$Q = q - \frac{k_2 a}{k_1 + k_2} \sin \omega t \ .$$

What is the Lagrangian in terms of Q? What is the corresponding Hamiltonian? Is the energy conserved? Is the Hamiltonian conserved?

Problem 3

Two people are holding the ends of a uniform plank of length L and mass m. What is the upward force required from each of the two people to hold the plank? Now, one person suddenly lets go, while the second person holds his/her end of the plank fixed. What is the upward force exerted by the second person immediately after the first person lets go? What is the downward acceleration of the free end of the plank immediately after the first person lets go?