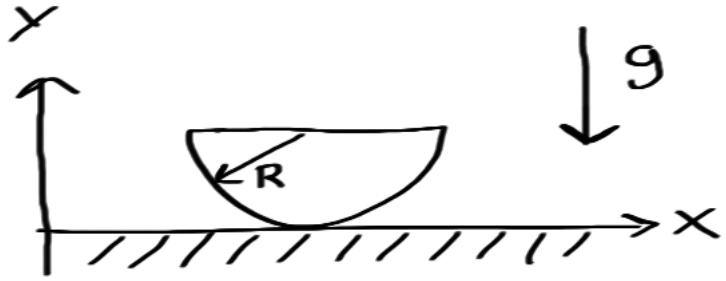
# Classical Mechanics

August 31, 2011

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

## Problem 1

A half-disk of radius $R$ and mass density $\rho$ (mass per unit area) can roll without slipping on flat surface in the $xy$-plane.




The total mass of the disk is

$$m = \int_{\text{disk}} \rho \, dx \, dy.$$

In the diagram the disk is shown in its equilibrium position. What is the frequency of small rolling oscillations of the disk with respect to equilibrium?

# Problem 2

Derive Snell's Law of refraction

# Problem 3

Assume that the Lagrangian for a system performing 1-dimensional motion is given by

$$L = e^{\gamma t}\left(\frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2\right)$$

where $\gamma$, $m$, and $k$ are positive constants.
a) [**2 points**] Are there any constants of motion?
b) [**3 points**] Determine q(t) and describe the motion? Describe all cases.
c) [**3 points**] Suppose a point transformation is made to another generalized coordinate S. given by

$$S = e^{\gamma t/2}q$$

What is the Lagrangian? What is the Lagrange equation? Are there any constants of motion? If so, derive their mathematical form.
d) [**2 points**] Describe the relationship between the two solutions. Especially comment on your findings for the constants of motion and how you reconcile any differences that you may find.