Classical Mechanics

August 29, 2012

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

A hypothetical material out of which an astronomical object is formed has an equation of state

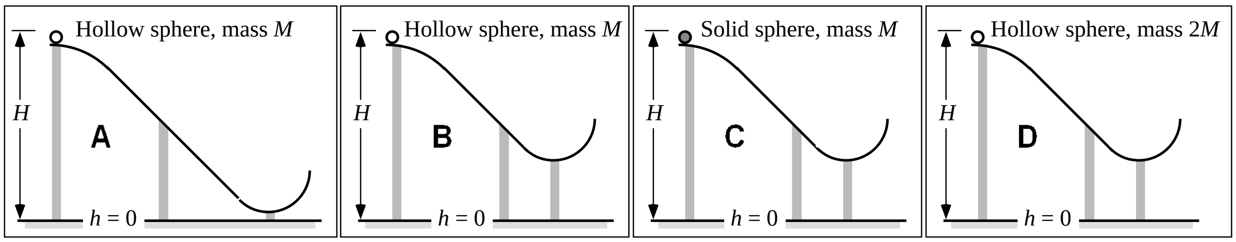
$$p = \frac{1}{2} K \rho^2$$

where p is the pressure and ρ is the density.
a) Show that, for this material, the density and the gravitational potential are linearly related under hydrostatic equilibrium conditions.
b) Use your results from part a to derive a differential equation for the density under hydrostatic equilibrium conditions. What boundary conditions or physical constraints should be applied?
c) Assuming spherical symmetry, find the radius of the astronomical object.
<u>Spherical</u> part of the Laplacian:

$$\Delta_r = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r}$$

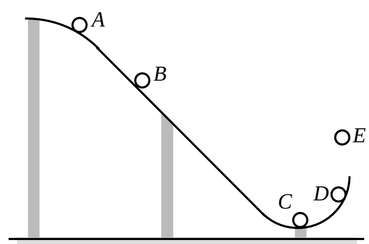
Problem 2

(a) In each case shown, a sphere is released from rest to roll without slipping down a track. The track is shaped so that the sphere is launched into the air vertically at the end of the track. The initial height of the spheres is the same (H), and the spheres all have the same radius r. The tracks are identical, except that the straight portion of the track in case A is longer. The sphere in case C is solid; all the others are hollow. The sphere in case D has a mass $2M$; all the others have a mass M. Ignore any dissipative forces.

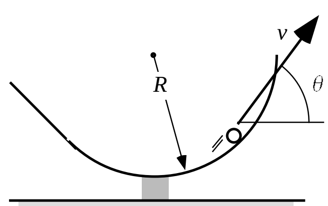


[3 points] Rank, from greatest to least, the maximum height that the spheres reach after they leave the track. If any spheres reach the same height, state that explicitly. Explain how you determined your ranking.

For the sphere in case A:
(b)[2 points] Show the approximate direction of the sphere's acceleration at the points indicated. If the acceleration is either perpendicular to the track or parallel to the track, state that explicitly. At point E, the sphere is moving upward.



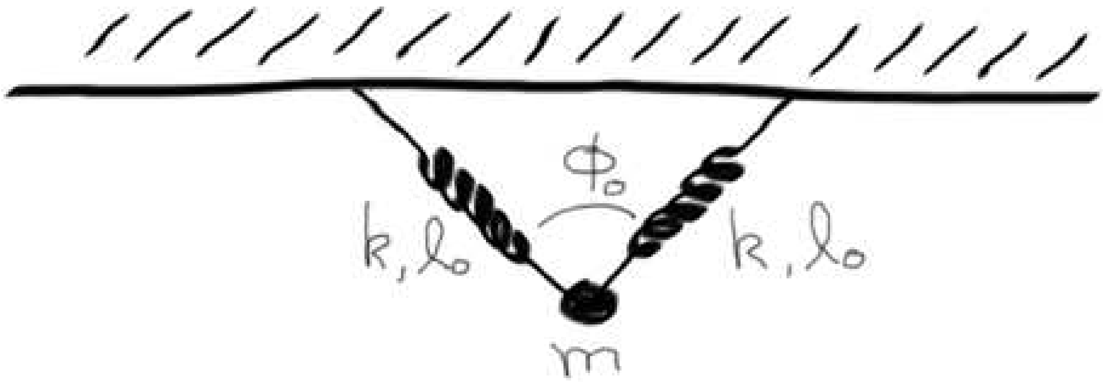
(c)(Below) [2 points] Draw a free-body diagram for the sphere in case A when it is at the position shown, with a velocity v at an angle θ to the horizontal. Label all forces, indicating the type of force and the object that is exerting that force.



(d)[3 points] Find the magnitude of each force in your free-body diagram. Assume the radius of the sphere is much smaller than the radius of curvature of the track R, Express your answer in terms of M, r, R, θ and g

Problem 3

A point mass m can move without friction on a flat horizontal surface. It is attached to two identical springs; the springs have the same elastic constant k and equilibrium length l_0. The ends of the springs are attached to a straight surface, as shown in the figure. The mass is shown at rest in the equilibrium position.



What are the frequencies of oscillation for small motions of the mass with respect to the equilibrium position?