Classical Mechanics

August 27, 2014

Work 2 of the 3 problems. Please put each problem solution on a separate sheet of paper and put your name on each sheet.

Problem 1

Consider a particle of mass, m, moving in a bound orbit with potential

$$V(r) = -\frac{k}{r}$$

Using polar coordinates (r, θ) in the plane of the orbit answer the following questions:
a) Find the radial and the angular momentum as functions of $r, \theta, \dot{r}, \dot{\theta}$. Is either of the momenta conserved?
b) Using the virial theorem ($2\bar{T} = n\bar{V}$; for $V = \alpha r^n$, where $\bar{T}$ and $\bar{V}$ are the average kinetic and potential energies for one complete orbit, respectively) show that

$$J_r + J_\theta = \oint \frac{k}{r} dt$$

where

$$J_r = \oint p_r dr$$

$$J_\theta = \oint p_\theta d\theta$$

c) Show that (E is the total mechanical energy of the system)

$$J_r + J_\theta = \sqrt{\frac{-2\pi^2 m k^2}{E}}$$

$$\int_{r_-}^{r_+} \frac{dr}{\sqrt{-r^2 + ar - b}} = \pi; r_\pm = \frac{1}{2}(a \pm \sqrt{a^2 - 4b})$$

d) Using the results of part c show that the period of the orbit τ is the same for r and θ motions, namely

$$\tau = \pi k \sqrt{\frac{m}{-2E^3}}$$

Problem 2

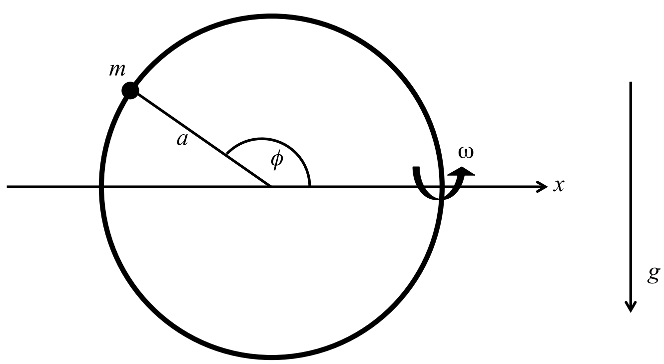
A bead of mass m slides without friction on a smooth circular wire of radius a that is rotating with constant angular velocity ω about a fixed horizontal diameter. At $t = 0$ the wire is in a vertical plane. Let ϕ be the angle that the radius drawn to the bead makes with the axis of rotation.

(a) Determine the kinetic and potential energies of the bead in terms of generalized coordinate(s).

(b) Determine the equation(s) of motion for this system.

(c) If the wire were not rotating (that is, the wire is always in a vertical plane), what would be the equation(s) of motion? Discuss the character of the motion.

(d) Now suppose the wire is rotating, and the experiment were carried out in a laboratory orbiting the earth. What would be the equation(s) of motion? Discuss the character of the motion.



Problem 3

A particle of mass m moves classically in a 1-dimensional potential $V(z) = \mathcal{C}|z|^k$, where $\mathcal{C}$ and k are positive constants.
a) For a given total energy, E, what is the average value of the potential energy $V(z)$?
b) Give the dependence of the period τ on the energy E up to a constant factor independent of E.