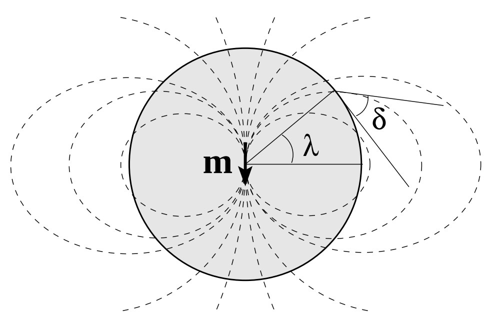
Electromagnetism

August 25, 2013

Work 4 (and only 4) of the 5 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

Assume that the Earth's magnetic field is the same as that of a small magnetic dipole situated at the center of the Earth with its axis through the geographical poles. Find the angle of dip, δ, between the magnetic field lines and the surface of the Earth at latitude λ. Calculate the angle of dip of the Earth's magnetic field lines in Las Cruces (latitude: $32.3°$). The latitude angle is measured from the equator.



Problem 2

A charge density ρ_0 is placed at time $t = 0$ in a small region in the interior of a homogeneous chage-neutral material that has electric conductivity σ.
a) Derive expressions for the time evolution of the charge density in that region, $\rho_c(t)$, with $\rho_c(0) = \rho_0$. Hint: Use a continuity equation.
b) Estimate how long it will take (in seconds) for the charge density to decrease to 1/1000 of its original (initial) value if the material is;
(i) copper with conductivity $\sigma = 1/(2\mu\Omega cm)$ and (ii) quartz with conductivity $\sigma = 1/(10^{24}\mu\Omega cm)$
. Use $\rho_0 = 8.85 \times 10^{-12} c^2/Nm^2$

Problem 3

Note: Please write text and equations neatly, preferably with a sharp, soft pencil. If I cannot read your work, I cannot give you credit. Read the assignment carefully and follow the instructions. There are 4 different questions numbered from 1 to 4. Answer each of them separately and clearly mark the part you are addressing with your answer.

An electromagnetic wave

$$\vec{E}(\vec{r}, t) = \vec{E}_0 \exp\left[i\left(\vec{k}\cdot\vec{r} - \omega t\right)\right] \tag{1}$$

with wave vector $\vec{k}$ and (angular) frequency ω travels through a solid homogeneous material with a frequency-dependent dielectric function $\epsilon(\omega) = \epsilon_1(\omega) + i\epsilon_2(\omega)$. This wave leads to a polarization $\vec{P}$ described by the complex susceptibility

$$\chi(\omega) = \frac{\omega_P^2}{\omega_0^2 - \omega^2 - i\gamma\omega}, \tag{2}$$

where ω_P, ω_0, and γ are materials constants (real quantities), which are called plasma frequency, resonance frequency, and damping rate, respectively. For a dielectric, $0 < \gamma \ll \omega_0$. For a metal, ω_0=0.

1. For a dielectric, calculate the phase shift between the electric field $\vec{E}$ and the polarization $\vec{P}$ in the limit $\omega \to 0$ and $\omega \to \infty$.

2. For a dielectric, calculate the phase shift between the electric field $\vec{E}$ and the polarization $\vec{P}$ for $\omega = \omega_0$.

3. If an electromagnetic wave propagates through a material (regardless of what it is), it does not make sense to distinguish between the polarization current density and the common current density $\vec{j}$. Assuming that

$$\vec{j} = \sigma\vec{E} = \frac{\partial\vec{P}}{\partial t}, \tag{3}$$

   express the complex conductivity $\sigma(\omega) = \sigma_1(\omega) + i\sigma_2(\omega)$ as a function of the complex susceptibility $\chi(\omega)$. Use complete sentences to describe the physical meaning of the real and imaginary part of the complex conductivity $\sigma(\omega)$.

4. The dissipated energy in the material is proportional to the imaginary part of the susceptibility. At what (angular) frequency ω does the wave reach maximal dissipation?

Hint: I am using the following conventions for the various electromagnetic fields:

$$\vec{D} = \epsilon\epsilon_0\vec{E} \tag{4}$$
$$\vec{D} = \epsilon_0\vec{E} + \vec{P} \tag{5}$$
$$\vec{P} = \epsilon_0\chi\vec{E} \tag{6}$$
$$\epsilon = 1 + \chi \tag{7}$$

Problem 4

A radiating electric dipole consists of a rod of length l with charge $+q$ at one end and charge $-q$ at the other end. The rod lies in the x, y plane and rotates about the z - axis with angular velocity ω. Calculate:

a) the dipole moment,

b) the angular distribution of the radiation power, $dP/d\theta$, and

c) the total radiation power P .

Problem 5

For a charge Q' placed a distance d from a conducting sphere of radius R, an image charge $q' = -\frac{R}{d}Q'$ placed a distance $r = \frac{R^2}{d}$ from the center of the sphere creates a potential that has the same value at all points just outside the surface of the sphere.

A point charge Q is placed a distance 2a from the center of a neutral, insulated conducting sphere of radius a.

a) What is the stored electrical energy in this system?

b) Find the electrical field at $z = 1.5a$ on the z-axis ($z = 0$ is in the center of the sphere).

c) How much charge would have to be added to the sphere so that the electric field at $z = 1.5a$ is zero? What is $E(z)$ along the Z-axis for $a < z < 2a$ in this case?

d) For the situation in (c), suppose an additional charge q is placed at $z = 1.5a$. What is the electric force on q?