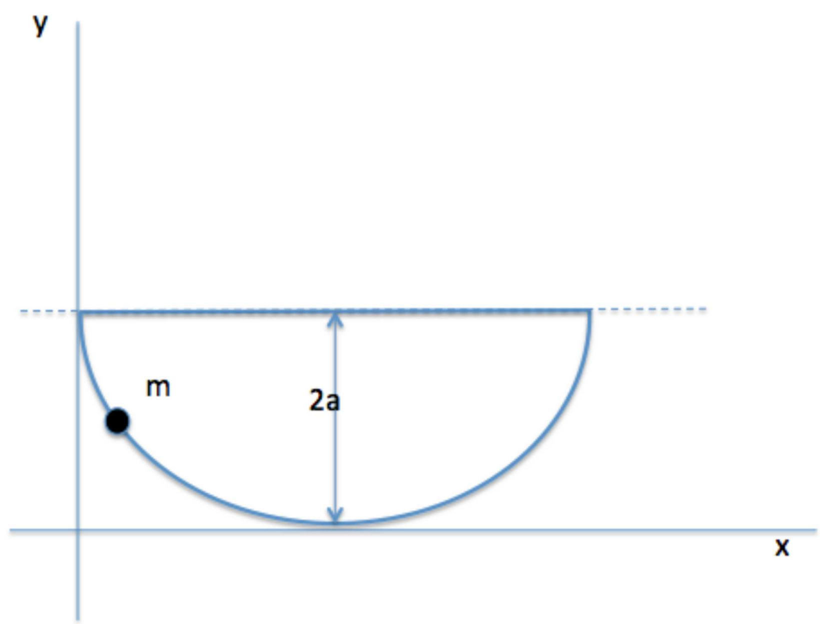
# Classical Mechanics

August 26, 2015

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

## Problem 1




A bead slides without friction on a frictionless wire in the shape of a cycloid with equations

$$\begin{aligned} x &= a(\theta - \sin\theta) \\ y &= a(1 + \cos\theta) \end{aligned}$$

where $0 \leq \theta \leq 2\pi$. Find: a) The Lagrangian function; b) The equation of motion.

# Problem 2

A particle of mass $m$ moves in a 1-dimensional potential $U(x) = A|x|^n$, where $A$ and $n$ are positive constants. Give the dependence of the period $\tau$ on the energy $E$ up to a constant factor independent of $E$.

# Problem 3

Poisson brackets provide some of the most fundamental insights into classical mechanics. They are useful in describing conserved quantities and also provide the classical analogue to the Heisenberg picture in quantum mechanics. The present problem is concerned with the former aspect.

a) [4 points] Prove the following relationship: If $f$ and $g$ are constants of motion, then the Poisson bracket $[f, g]$ is also a constant of motion. In other words, show: $f, g = constant \Rightarrow [f, g] = constant$.

b) [4 points] Show that $L_z$, the $z$-component of the angular momentum, is a constant of motion if $L_x$ and $L_y$ are constants of motion.

c) [2 points] Justify the following true statement: In a planetary system in which all planets are moving in the $z = 0$ plane, the $z$-component of the angular momentum of each planet is a constant of motion regardless of the interaction between the planets. Why do the $z$-components of the individual angular momenta of the Earth and the Moon nevertheless change continuously in reality?

Some possibly useful relationships **(do not prove)**:

Jacobi identity for Poisson brackets:

$$[f, [g, h]] + [g, [h, f]] + [h, [f, g]] = 0$$

The completely antisymmetric tensor of rank three is defined as

$$\epsilon_{ikl} = \begin{cases} 1 & \text{for } ikl \text{ an even permutation of } 123 \\ -1 & \text{for } ikl \text{ an odd permutation of } 123 \\ 0 & \text{else} \end{cases}$$