Electrodynamics

August 24, 2015

Work 4 (and only 4) of the 5 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

A charged parallel-plate capacitor with a uniform electric field is placed in a uniform magnetic field pointing parallel to the capacitor plates (the capacitor plates are parallel to the xy plane). The spacing between the plates is d and the area of the plates is A.

a) Find the **electromagnetic field momentum** in the space between the plates. How do you explain that there is momentum if nothing is moving?

b) Consider now a resistive wire connecting the two plates along the z axis, so the capacitor slowly discharges. The current through the wire will experience a magnetic force; what is the total impulse (change in momentum) delivered to the system during the discharge?

Problem 2

A point particle of mass m and magnetic dipole moment M moves in a circular orbit of radius R about a fixed magnetic dipole (moment M_0), located at the center of the circle. The vectors $\overrightarrow{M_0}$ and $\overrightarrow{M}$ are antiparallel to each other and perpendicular to the plane of the orbit.

Questions:

a) Compute the magnitude v of the velocity of the orbiting magnetic dipole.

b) Is the orbit stable against small perturbations? (Consider only the motion in the plane)

Problem 3

A dielectric sphere with dielectric constant K of radius R has a free charge density ρ distributed uniformly throughout the volume.

a) What is the electrostatic potential at the center of the sphere, relative to infinity?

b) How much energy is required to establish this configuration, starting with the charge dispersed at infinity?

Problem 4

Four shorter questions:

a) In 1923, Compton performed a series of experiments in which he was scattering X-rays from a graphite scatterer. The wavelength of the X-rays emitted by his source was $\lambda = 0.7$ Angstroms. What was the wavelength of the longest wavelength scattered X-rays that he observed?

b) Suppose a small sphere of charge $-q$ is suspended along the positive $\hat{z}$-axis by a massless string above a very large plate of charge Q and area A (where A is much larger than the distance from the charged surface to the sphere). Suppose the mass is moving horizontally with a velocity $\vec{v} = -v_0\hat{x}$ with respect to the plate. What is the electromagnetic force acting on the sphere in the reference frame of the sphere? You should NOT assume that $v_0 \ll c$, but you may ignore any radiative losses.

c) Demonstrate that Maxwell's equations $\vec{\nabla} \times \vec{E} = -\partial\vec{B}/\partial t$ and $\vec{\nabla} \cdot \vec{B} = 0$ are compatible, i.e., the first one does not contradict the second one.

d) Imagine an electric charge moving in the field of a magnetic monopole (although none has yet been found). Set up the non-relativistic equation of motion for an electric charge q of mass m in the field of a magnetic monopole of strength Γ (a positive constant). Assume the particle at a particular moment in time is a distance r from the magnetic monopole and that the particle's velocity $\vec{v}$ is perpendicular to the line between the charged particle and the monopole. Give an expression for the force vector on the particle at this point.

Problem 5

An electromagnetic wave with angular frequency ω propagates along the z-axis through a non-conductive, non-magnetic medium which is described by the polarization vector

$$\vec{P} = \gamma \vec{\nabla} \times \vec{E},$$

where γ is a (real) constant and $\gamma c \mu_0 \omega \ll 1$. Starting with Maxwell's equations,

a) Show that the wave sees two different refraction coefficients and derive these coefficients in terms of γ, ω, and fundamental physical constants. (7 points)

b) Identify the components of the wave that see each refraction coefficient. (3 points)