Classical Mechanics

August 20, 2016

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

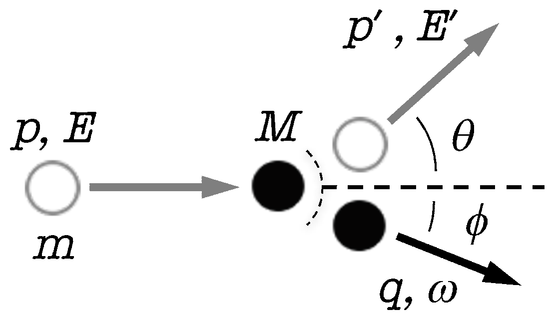
Problem 1

Two particles move around each other in circular orbits under the influence of gravity with a period T. Their motion is suddenly stopped and they are allowed to crash into each other. Show that they collide after a time $T/(4\sqrt{2})$.

Problem 2

Consider a spherical pendulum consisting of a point mass on a rigid massless rod connected to the origin in a gravitational field. Find the equations of motion for the pendulum (as a function of angles θ and ϕ; r is constant). Do **not** solve them.

Problem 3



Consider the figure above and work in the *laboratory* system. A *non-relativistic* particle of mass m with energy E and momentum p collides with a stationary target of mass M. The incident particle scatters with energy E' (momentum p') at an angle θ. The target particle recoils with energy ω (momentum q) at an angle ϕ.

a.) Conceptually, no matter what masses are used, can the target particle (mass M) recoil *past* 90°? Explain your answer.

b.) Similarly, if $m > M$, can the incident particle be scattered *past* 90°? Explain your answer.

c.) Calculate the target recoil momentum q in terms of the incident momentum p and the target scattering angle ϕ.

d.) Similar to the previous question, show that the target recoil energy ω in terms of the incident energy E and the target scattering angle ϕ is

$$\omega = \frac{4mM}{(m+M)^2} E \cos^2 \phi$$

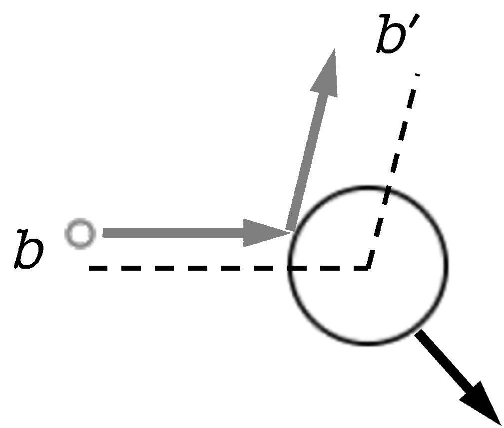
Now, consider the case when $m = M$.

e.) Calculate the recoil energy ω in terms of the incident particle energy E and *now* the outgoing particle direction θ.

Low-energy neutrons engage in non-relativistic elastic collisions with atomic *nuclei*. We design neutron shields with materials that can absorb the most energy per elastic collision. Therefore, f.) What nucleus is the best to use to reduce the energy of a neutron? What types of materials are rich in this nucleus? Your first, most obvious guess for material may not satisfy practical engineering concerns, why?

Suppose we set up an experiment to measure ω and find the number of events per energy bin is flat, i.e., $dN/d\omega = \text{const}$.

g.) What does this tell you about the recoil angular distribution $dN/d\cos\phi$?



Finally, real particles have physical size and collisions are rarely *head-on*. The *impact parameter* b measures how far off the central axis the incident particle is. The same is true that the scattered particle trajectory will be parallel to but displaced by b' to a vector from the center of the target particle.

h.) If the origin is at the center of the target particle, then what is the incident angular momentum?

i.) The figure shows $b' > b$. Can you explain why this is the case?