Electrodynamics

August 19, 2016

Work 4 (and only 4) of the 5 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

A point particle of mass m and magnetic dipole moment M moves on a circular orbit of radius R about a fixed magnetic dipole, moment M_0, located at the center of the circle. M and M_0 are antiparallel and oriented perpendicular to plane of the orbit.

a.) Compute the velocity v of the orbiting dipole.

b.) Is the orbit stable against small in-plane perturbations? Explain.

Problem 2

An infinitely long cylindrical tube, radius a, moves at constant speed v along its axis. It carries a net charge per unit length λ, uniformly distributed over its surface. Surrounding it, at a radius b, is another cylinder, moving with the same velocity, but carrying the opposite charge $(-\lambda)$. Find:

a.) the energy per unit length stored in the fields.

b.) the momentum per unit length stored in the fields.

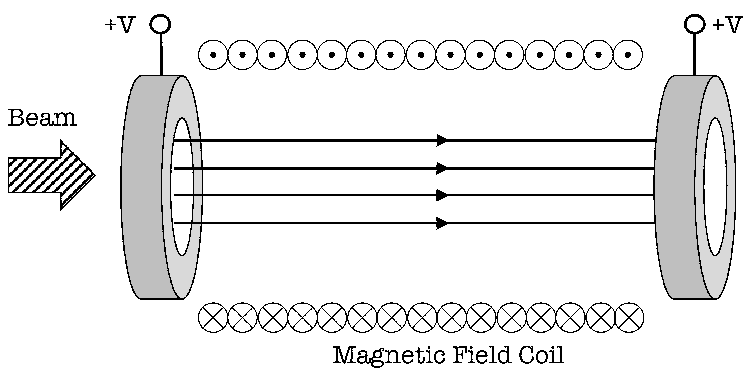
c.) the energy per unit time transported by the fields across a plane perpendicular to the cylinders.

Problem 3

To perform precision atomic and nuclear physics measurements, it is useful to "trap" charged particles for further study. Loosely, we consider a particle *trapped* if there is some restoring force that keeps the particle away from the "walls" of the apparatus.

a.) Consider a hollow, positively-charged, sphere with a proton placed at the center. Explain why this sphere *does* or *does not* trap the proton.

An alternative way to trap charged particles is by using a *Penning* trap as shown in the figure below. A Penning trap uses a magnetic field to constrain motion in the $\hat{x} - \hat{y}$ axes, and electrostatic repulsion to constrain motion in the $\hat{z}$ axis. In the figure below, the electrostatic field is set up by a pair of annuli to allow radioactive beams to penetrate and decay to charged particles at the center of the trap.



b.) Assume that the length of the trap is $\ell = 20$ cm and within this length there are $N = 1000$ windings. If you ignore edge effects, what is the current I that is required to produce a 1 Tesla magnetic field at the center of the trap?

c.) Assume that the diameter of the magnetic field coils is $D = 6$ cm, and copper wire with radius $r = 1$ mm is used. If the resistivity of copper is $1.7 \times 10^{-8}\ \Omega \cdot$m, then calculate the following:

- Total wire resistance R.
- Voltage V required to yield 100 A of current.
- Total power P at 100 A of current.

d.) Model the annulus as a ring of charge Q with radius R. What is the functional form of the electric field $E(z)$ along the beam axis $\hat{z}$?

e.) If the ring of charge is 4 cm in diameter, how much charge Q is required to have a potential of 1000 V at the center of the ring?

f.) Consider a neutron decay in the Penning trap that produces a 700 eV proton (mass = 938.3 MeV/c^2 = 1.673×10^{-27} kg). If the magnetic field is 5 T, then what is the maximum cyclotron radius possible for this proton?

The Lorentz force law is

$$F = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \ .$$

The equations of motion are complicated when $\vec{E} \perp \vec{B}$. In this case, the motion is still cyclotron orbits, but now the guiding center of the cyclotron orbit drifts with velocity $\vec{v}_d$.

Therefore, substitute $\vec{v} = \vec{v}_\perp + \vec{v}_d$, and require only rotational motion in this frame of reference,

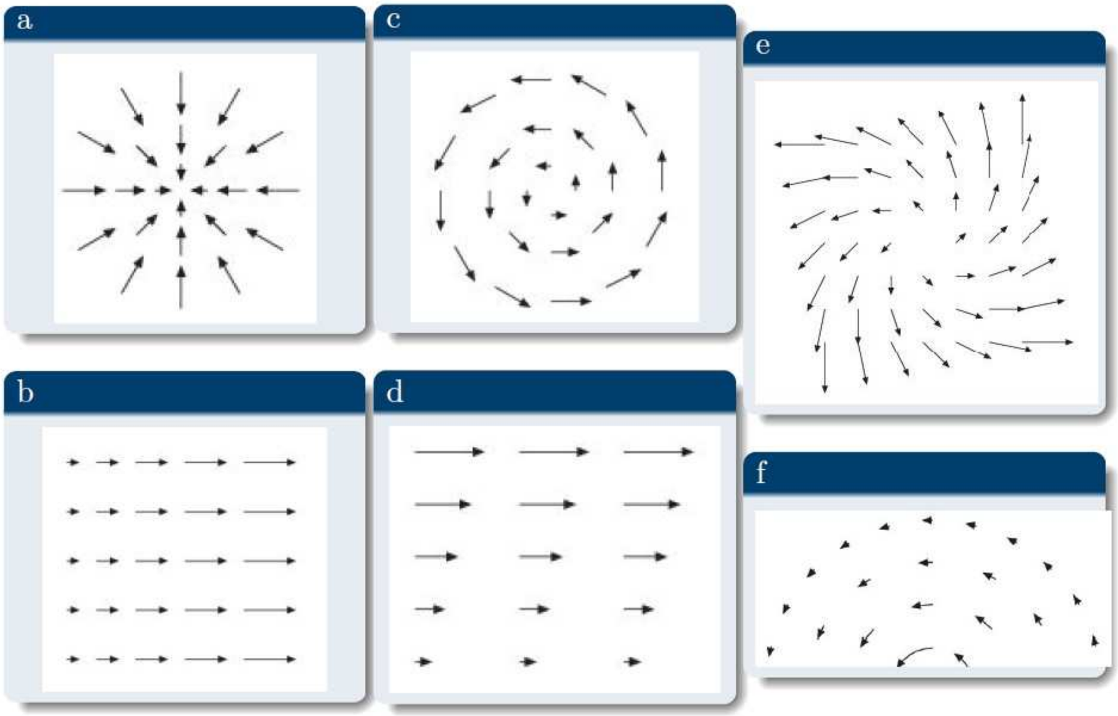
$$\begin{aligned} F &= q\left(\vec{E} + \vec{v}_\perp \times \vec{B} + \vec{v}_d \times \vec{B}\right) \\ &\rightarrow q\left(\vec{v}_\perp \times \vec{B}\right) \\ \text{when} \quad & \vec{E} = -\vec{v}_d \times \vec{B}. \end{aligned}$$

g.) Using the last constraint, calculate the vector $\vec{v}_d$ as a function of $\vec{E}$ and $\vec{B}$ when $\vec{E} \perp \vec{B}$. Assume that there is no component of $\vec{v}$ parallel to $\vec{B}$, so that we are dealing with motion in a plane perpendicular to $\vec{B}$. *Hint*: It may be helpful to assume $\vec{E} = |E|\hat{y}$ and $\vec{B} = |B|\hat{z}$ without loss of generality at intermediate steps, but give a coordinate-independent final answer.

Notes: Permeability of vacuum $\mu_0 = 4\pi \cdot 10^{-7}\,\mathrm{N/A^2}$; permittivity of vacuum $\epsilon_0 = 1/(\mu_0 c^2)$; unit of charge 1C = 1As; unit of magnetic field 1T = 1kg/(As^2); unit of electric potential 1V = 1Nm/C; electron charge $e = 1.6 \cdot 10^{-19}$ C.

Problem 4

a.) Consider the six depicted vector fields in two dimensions,



Which of the six vector fields can<u>not</u> be written as the gradient of a scalar field? Hint: it is more than one! Briefly explain your answer. Your explanation should include drawing on the vector field depictions. (3 points)

b.) Imagine a superconducting ring. At $t \ll 0$ no current flows through the ring. At $t = 0$ a hypothetical magnetic monopole passes through the ring. Qualitatively, sketch the magnetic current flowing through the ring as a function of time. Explain your answer. (3 points) c.) A particle is at rest until it suddenly starts moving with velocity $v_z < c$. Qualitatively draw the electric field lines around the particle at time $t > 0$ in the x-z plane. Indicate the position vt of the particle at time t in your plot. Be sure to distinguish between distances less than ct and greater than ct from the origin when drawing the field lines. (4 points)

Problem 5

An optically active medium can rotate the plane of polarization of light. The susceptibility tensor of such a medium can be expressed as:

$$\bar{\chi} = \begin{pmatrix} \chi_{11} & i\chi_{12} & 0 \\ -i\chi_{12} & \chi_{11} & 0 \\ 0 & 0 & \chi_{33} \end{pmatrix}$$

where $\bar{\chi}$ is related to the polarizability tensor in the usual fashion. $\vec{P} = \epsilon_0 \bar{\chi} \cdot \vec{E}$ and χ_{11}, χ_{22}, χ_{33} are real. Assume a plane wave propagates in this medium along the z-direction (which is also the 3-direction) with frequency ω. Use Maxwell's equation to establish the following:

a.) That in an optically active medium the propagating electromagnetic wave is transverse.

b.) Show that the medium admits electromagnetic waves with two distinct k-vectors $\vec{K}_R$, $\vec{K}_L$. Find the magnitudes of $\vec{K}_R$, $\vec{K}_L$ in terms of ω and the necessary χ_{ij}.

c.) Show that the two k-vectors $\vec{K}_R$, $\vec{K}_L$ correspond to the propagation of right- and left-handed circularly polarized electromagnetic waves.

d.) Find an expression for the rotary power $\equiv n_R - n_L$ in terms of the χ_{ij}, where n_R, n_L correspond to the refractive indices of the medium for the propagation of right- and left-handed circularly polarized light.