Statistical Mechanics

August 19, 2017

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

The so-called troposphere, the lower 10–15 km of the atmosphere, cannot be considered as isothermal. A reasonable model of the troposphere is to describe it as a convective steady state at constant entropy, where Pv^γ is independent of the altitude and $v \equiv V/N$ is the specific volume of the gas (adiabatic-atmosphere model).

Assume that the atmosphere is composed entirely of diatomic N_2 molecules, which can be treated as an ideal gas.

a.) Show that dT/dP at any altitude in the troposphere obeys the following relationship,

$$\frac{dT}{dP} = \left(\frac{\gamma - 1}{\gamma}\right)\frac{T}{P}$$

b.) Show that the temperature gradient dT/dz as a function of altitude in the troposphere is constant. *Hint: Use the chain rule to relate dT/dz to dP/dz. Your final answer for dT/dz should depend on γ, $m(N_2)$ and g.*

c.) What is the value for γ that you should use for the troposphere? Explain.

d.) Compute the temperature difference between sea level and the top of Mount Everest, which is at an altitude of 8848 m.

Values and constants that you may need:

$g = 9.8\,\mathrm{m/s^2}$

$k = 1.38 \cdot 10^{-23}\,\mathrm{J/K}$

$m(N_2) = 4.653 \cdot 10^{-26}\,\mathrm{kg}$

Problem 2

Consider a gas of photons in thermal equilibrium at temperature T in a *one-dimensional* cavity of length L.

a.) Calculate the density of states $g(\omega)$.

b.) Find the internal energy E and specific heat C_V.

c.) Find the entropy S, Helmholtz free energy A, and pressure P.

$$\left(Hint: \int_0^\infty \frac{x dx}{e^x - 1} = \frac{\pi^2}{6} \right)$$

Problem 3

A mole of ^{3}He gas atoms has a volume of $0.0224\,\mathrm{m}^3$ at 273K. The mass of a ^{3}He gas atom is $5.11 \cdot 10^{-27}$ kg and it has spin $\frac{1}{2}$. Calculate the value of $\exp(-\mu/kT)$, where μ is the chemical potential of the atoms, and determine the mean occupancy of a single particle state of energy E.