Classical Mechanics

August 18, 2018

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

Assume the Lagrangian for a certain one-dimensional motion is given by

$$L = e^{\gamma t}\left(\frac{1}{2}m\dot{q}^2 - \frac{1}{2}kq^2\right)$$

where γ, m, k are positive constants.

Questions:

a.) What is Lagrange's equation?

b.) Are there any constants of motion?

c.) How would you describe the motion?

Suppose a point transformation is made to another generalized coordinate, S, given by

$$S = \frac{e^{\gamma t/2}}{2}q$$

Questions:

d.) What is the new Lagrangian?

e.) Are there any constants of motion?

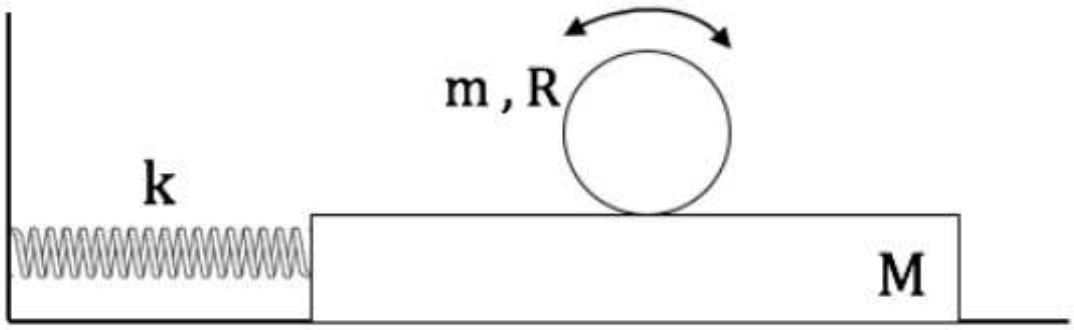
f.) Describe the relationship between the two solutions.

Problem 2

A brick falls flat onto a tennis ball resting on the ground and bounces back to the height of h. What height does the ball bounce to?

Problem 3

A block of mass M slides along a frictionless surface. The block is connected to the wall with a spring of constant k. A cylinder of mass m, radius R and rotational inertia $\frac{1}{2}mR^2$ rolls without slipping on the block.



a.) In words, what type of motion occurs if displacement from the state of rest is small?

b.) What is the frequency of the motion in a.)?

c.) What is the maximum oscillation amplitude that the block can have before the cylinder starts slipping if the friction coefficient is μ?