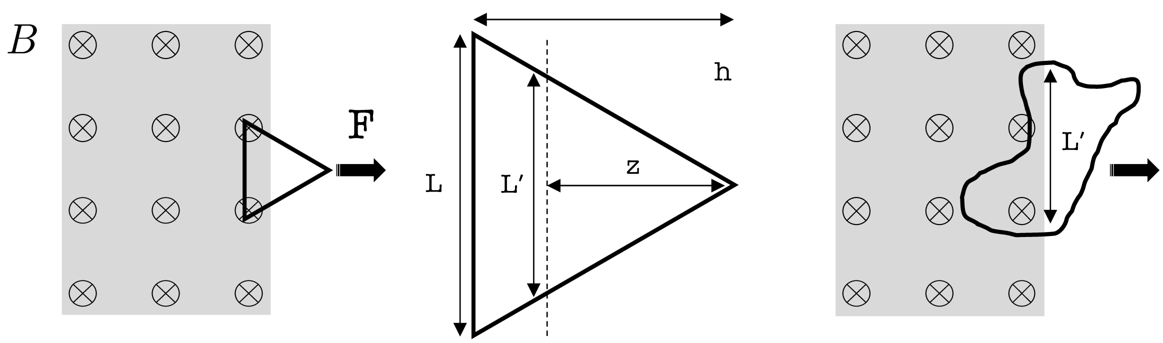
Electrodynamics

August 17, 2018

Work 4 (and only 4) of the 5 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1



You pull a triangular conductor horizontally through a uniform magnetic field B (left image). Outside the boundary the magnetic field is zero. The triangular loop has electrical resistance R, width L, and height h (middle image).

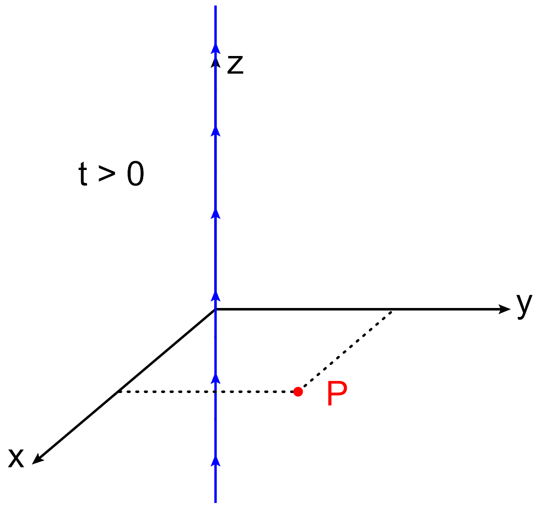
a.) (6 Points): If you wish to maintain a constant velocity v, what is the force $F(z)$ as a function of length z that has emerged from the magnetic field?

b.) (4 Points): Given what you've learned for a specific case, derive a formula for the force required to pull an arbitrary shape with resistance R moving at a constant velocity v in terms of L' (right image).

Problem 2

A photon moves along the z axis in the laboratory frame where it encounters an object of mass M_1 at rest. Out of this collision comes an object of mass M_2 and a photon moving in the x-z plane at an angle θ with respect to the z-axis. Find the relation between the wavelength of the incoming photon λ_1, and the wave length of the outgoing photon λ_2, and the two masses. The speed of light c and Planck's constant will enter this relation.

Problem 3



The figure shows a thin wire along the z-axis. A time-dependent current flows within the wire. We have a current density of the form

$$\vec{\jmath}(\vec{x},t) = I_0 \delta(x)\delta(y)\vec{e}_z \begin{cases} 0 & \text{if} \quad t \leq 0 \\ t/\tau & \text{if} \quad t > 0 \end{cases} \quad ,$$

where I_0 and τ are positive constants.

a.) (2 Points): Show that the charge density $\rho(\vec{x},t)$ is time-independent.

b.) (6 Points): Now we assume that the wire is uncharged such that $\rho(\vec{x},t) \equiv 0$. Compute the *retarded potentials* $\varphi(\vec{x},t)$ and $\vec{A}(\vec{x},t)$ in an arbitrary location P in the x-y plane.
Hint:

$$\int_{-z_0}^{z_0} \frac{dz}{\sqrt{a^2+z^2}} = 2\ln\left(\frac{\sqrt{z_0^2+a^2}+z_0}{a}\right) \quad .$$

c.) (4 Points): Compute the electromagnetic fields $\vec{E}(\vec{x},t)$ and $\vec{B}(\vec{x},t)$ at the location P. Show that $|\vec{E}| \ll c|\vec{B}|$ at very large times t. Do these fields radiate energy?

Problem 4

Two identical classical solid spheres, radius R, each carrying a total charge q distributed uniformly throughout them, spin with an angular frequency ω. They are separated by a distance $r \gg R$. Their spin axes are parallel and aligned with the z-axis, while the line between the spheres is along the x-axis.

a.) What is the dipole magnetic moment $\vec{m}$ of either sphere?

b.) What is the (dipole) magnetic field at the position of one sphere due to the second sphere? Express your answer in terms of $\vec{m}$, the separation of the spheres, and any other relevant variables.

c.) What is the force on one sphere due to the other? Express your answer in terms of $\vec{m}$, the separation of the spheres, and any other relevant variables.

d.) What is the electromagnetic interaction energy of the two spheres? Express your answer in terms of $\vec{m}$, the separation of the spheres, and any other relevant variables.

Problem 5

An electromagnetic wave with frequency ω propagates through a medium containing N molecules per unit volume. In each molecule, an electron (mass m, charge $q = -e$) is bound by a force which can be described approximately as a spring force with constant $k = m\omega_0^2$; any other electrons are ignored in this problem as is any motion of the heavy nucleus. (Hint: It is convenient to use a complex expression for the electromagnetic wave, with the understanding that the physical fields are given by the real part.)

a.) Obtain a solution for the equation of motion of the electron and from that the electric dipole moment of the molecule, as a function of time, resulting from the motion of this electron. (2 points)

b.) From the result above, derive an expression for the polarization of the material and use it to calculate the dielectric constant. Sketch the index of refraction as a function of ω around $\omega = \omega_0$. (3 points)

c.) A more realistic treatment will also include a "damping" term which becomes important near ω_0. To a good approximation, this can be described by a force proportional to the velocity, $F_d = -\gamma m\dot{x}$, where $x(t)$ is the displacement of the electron due to the electric field. Repeat the calculations above and find a modified expression for the dielectric constant; the equation for the electromagnetic wave should now have an absorptive part as well. Again sketch the refractive index as well as the coefficient of absorption around ω_0. (3 points)

d.) The refractive index in part c.) should have a minimum and a maximum on either side of ω_0. Show that these occur at the values of ω where the absorption coefficient is half its maximum value. Assume $\gamma \ll \omega_0$. (2 points)