Statistical Mechanics

August 18, 2018

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

A classical ideal gas composed of N identical particles of mass m is confined to a *two-dimensional* disk of radius R. The gas particles are attracted to the center of the disk by a force that increases proportionally to the distance from the center. The Hamiltonian of a single particle is given by

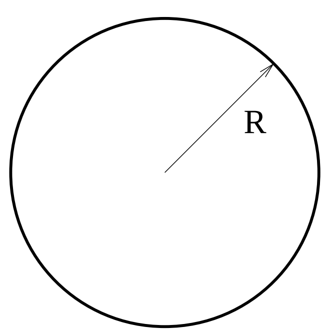
$$H = \frac{p^2}{2m} + \frac{\kappa r^2}{2}, \tag{1}$$

where κ is the effective spring constant of the force and r is the distance of the particle from the disk's center.

a.) Find the canonical partition function of this system.

b.) Calculate the internal energy E, Helmholtz free energy A, and pressure P of this gas.

c.) Evaluate E and P in the limits of (i) $\kappa \to 0$ and (ii) $R \to \infty$.



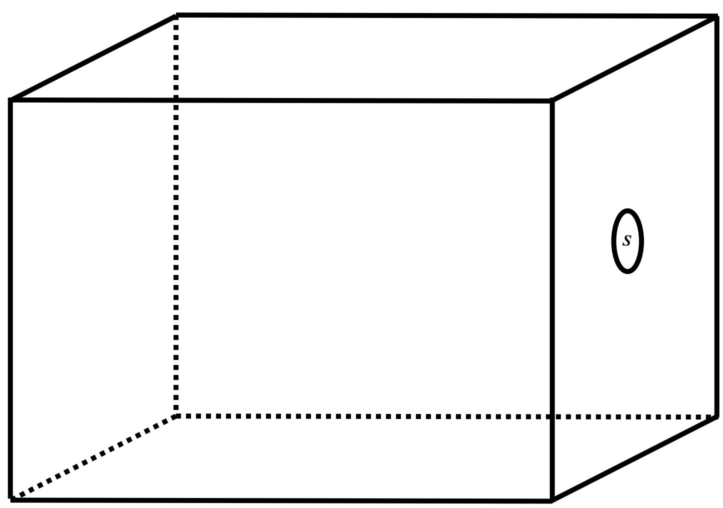
Problem 2

a.) The low-density occupation function applicable to an ideal gas of N particles is

$$f(\epsilon) = \exp[(\mu - \epsilon)/kT] \ ,$$

where $f(\epsilon)$ is the occupancy of a single-particle quantum state of energy ϵ, μ is the chemical potential, and T is the temperature. Starting from this, derive the probability distribution function for particle speed, $P(v)$, for a spinless monatomic ideal gas.

b.) A container of volume V holds N atoms of a spinless monatomic ideal gas; the container is maintained at a fixed temperature T. Imagine a small hole of area S on the wall; calculate how many particles will pass out of the hole per unit time.



Information:

The chemical potential of a three-dimensional ideal gas is $\mu = kT \ln[N/n_Q V]$, where $n_Q = (mkT/2\pi\hbar^2)^{3/2}$ is the quantum concentration.

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2} \qquad \int_0^\infty x e^{-x^2} dx = \frac{1}{2}$$

$$\int_0^\infty x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4} \qquad \int_0^\infty x^3 e^{-x^2} dx = \frac{1}{2}$$

Problem 3

The energies of an oscillator with frequency ν are given by

$$\epsilon = \frac{1}{2}h\nu, \frac{3}{2}h\nu, \ldots, \left(n + \frac{1}{2}\right) h\nu, \ldots$$

Consider a system containing N oscillators that can be treated as approximately independent.

a.) When the system has the total energy

$$E = \frac{1}{2}Nh\nu + Mh\nu \ ,$$

where M is an integer, find the corresponding statistical weight $W(M)$.

b.) Determine the relation between the temperature and the energy of this system.