Statistical Mechanics

August 24, 2019

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

A gas is described by this equation of state:

$$\left(p + a\frac{N^2}{V^2}\right)(V - Nb) = Nk_BT$$

where N is the number of particles of the gas, V is the volume, p is the pressure, T is the Kelvin temperature, k_B is the Boltzmann constant, and a and b are two positive constants. A sample of N particles of this gas is taken through a three-step thermodynamic cycle:

i.) constant volume transformation from (p_1, V_1, T_1) to (p_2, V_1, T_2); $p_2 > p_1$

ii.) constant temperature transformation from (p_2, V_1, T_2) to (p_1, V_2, T_2); $V_2 > V_1$

iii.) constant pressure transformation from (p_1, V_2, T_2) to (p_1, V_1, T_1)

Each step is performed reversibly.

a.) Sketch this thermodynamic cycle on a pV diagram.

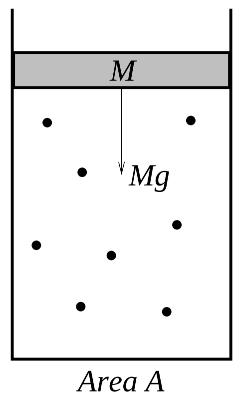
b.) Calculate the work done on the gas during this cycle.

Problem 2

A classical monatomic ideal gas composed of N identical particles of mass m is enclosed in a vertical cylindrical vessel of cross-sectional area A. The top of the vessel is closed by a movable piston of mass M. The vessel is placed in a uniform gravitational field g. Assume that $N \gg 1$ and $M \gg Nm$.

a.) Find the canonical partition function of the system consisting of the gas and the piston.

b.) Calculate the internal energy and specific heat of this system.



Problem 3

An ionic solution fills the space between the electrodes of a parallel plate capacitor having a certain potential difference V_0. The capacitor, once charged, is disconnected from its potential source. Obtain an expression for the space charge distribution which exists after the system has reached thermal equilibrium. Assume, for the sake of simplicity, that the applied potential difference is so small that $eV_0 \ll kT$.