Classical Mechanics

August 22, 2020

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

Consider a pendulum which moves in a two-dimensional vertical plane, consisting of a point-like bob of mass m connected to a fixed point by a connector of negligible mass and length $l(t)$ which can be adjusted as a function of time by someone operating the device. Construct the Lagrangean and the Hamiltonian of the system. For general $l(t)$, does the Hamiltonian equal the energy and is it conserved? Show that the Hamiltonian equations of motion read

$$\dot{\varphi} = \frac{p}{ml^2} \qquad\qquad \dot{p} = -mgl\sin\varphi$$

where φ is the angle between the plumb line from the fixed point and the direction of the bob, as seen from the fixed point; p is the corresponding canonical momentum. Suppose now that the operator wants to achieve motion with a constant $\dot{\varphi} = \omega$. Use the relation between p and l which the first equation then implies to simplify and solve the second equation and thus obtain $l(t)$, given the length $l_0 = l(\varphi = 0)$ which the connector assumes at $\varphi = 0$. Under what condition on l_0, g and ω can the operator succeed in achieving the desired motion?

Problem 2

You are given a rigid, solid object of unknown composition. It has an irregular shape; there are no obvious symmetry axes. Your job is to find, experimentally, the principal axes of the object and the associated moments of inertia. You are not allowed to do any destructive testing of the object. However, you may attach strings to the surface of the object and suspend it; you may throw it in the air, you can make paint marks on the surface; and so on. Just don't cut it open or break it apart or anything like that.

Describe an experiment, or sequence of experiments, that will determine the principal axes and associated moments of inertia. Provide mathematical support for the steps you take.

Problem 3

Consider a particle of mass m subject to a central, attractive, Yukawa-type force

$$F = -\frac{k}{r^2} e^{-r/a},$$

moving in a circle of radius R.

a.) Determine the condition for R in order for the circular motion to be stable

b.) Compute the frequency of small, radial oscillations around this circular motion