Electrodynamics

August 21, 2020

Work 4 (and only 4) of the 5 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

A linearly polarized electromagnetic wave is incident at an angle θ on an infinitely extended plane made from a perfect conductor. The electric field is orthogonal to the plane of incidence. Assume the $x - z$ plane as plane of incidence, i.e. the $\vec{E}$ field is polarized in y direction. Also, let the conductor surface be at $z = 0$.

Find the charge and current densities induced on the conducting plane.

Hint: *Inside the perfect conductor, electric and magnetic fields vanish. Therefore, the boundary conditions for the fields right above the conductor's surface are:*

$$(i)\quad \vec{E}_{||} = 0 \qquad\qquad (ii)\quad E_{\perp} = \sigma/\epsilon_0$$

$$(iii)\quad B_{\perp} = 0 \qquad\qquad (iv)\quad (\vec{B} \times \vec{n})_{||} = \mu_0 \vec{K}$$

Problem 2

A non-relativistic positron of charge e and velocity v_1 ($v_1 \ll c$) impinges head-on on a fixed nucleus of charge Ze. The positron, which is coming from far away, is decelerated until it comes to rest and then is accelerated again in the opposite direction until it reaches a terminal velocity v_2. Taking radiation losses into account (which are assumed to be small) find v_2 as a function of v_1 and Z. What is the angular distribution and polarization of the emitted radiation?

Hints:

(i) Power of emitted radiation: $P = \frac{2e^2}{3c^3}\ddot{r}^2$, with the speed of light c and the radial acceleration $\ddot{r}$.

(ii)
$$\int \frac{1}{x^3\sqrt{x(x-x_0)}}dx = \frac{2\sqrt{x(x-x_0)}(8x^2+4xx_0+3x_0^2)}{15x^3x_0^3}$$

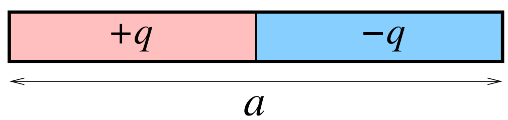
Problem 3

A thin rod of length a has a uniform distribution of positive charge $+q$ on one half and negative charge $-q$ on the other half. The rod lies in the $x-y$ plane and rotates about the z-axis with angular frequency ω.

a.) Find the time-dependent electric dipole moment produced by the rotating rod.

b.) Calculate the time-averaged power radiated per unit solid angle in the far zone.

c.) Determine the total power radiated by the rod.



Problem 4

You are presumably aware of the classic electrostatic image charge problem in three dimensions: A point charge q at a distance d above an infinitely extended, grounded conducting plane, with the former inducing a surface charge density σ on the latter; σ to be determined (you don't need to do this). Discuss this problem instead in two dimensions, where the conductor now becomes an infinitely extended line. Determine the analogous line charge density σ. Note that the electric field of a point charge in two dimensions is determined by the two-dimensional Gauss law.

Problem 5

a.) You observe a wire at rest, with a positively charged particle with charge Q flying with (a nonzero but non-relativistic) velocity v parallel to the wire at a distance R to the wire. The wire is electrically neutral, but a current I is flowing through the wire. Let's call the direction of the velocity and the direction of the current flow the $+\hat{z}$ direction.

i.) What is the electric field (magnitude and direction) at the position of the charged particle? (1 point)

ii.) What is the magnetic field (magnitude and direction) at the position of the charged particle? (1 point)

iii.) What is the direction of the force on the positively charged particle if the current flows in the same direction as the velocity of the particle? (1 point)

iv.) What is the magnitude of the force? (1 point)

b.) Assuming that the speed of the charge Q is nonrelativistic, when you compare the rest frame of the wire with the rest frame of the charge Q, how should the force from the wire on the charge Q compare between these two frames?(1 point) c.) In the rest frame of the charge Q, is there a magnetic force on the charge Q? (1 point)

d.) In the rest frame of the charge Q, is there an electric force on the charge Q? (2 points)

e.) Qualitatively explain how the force in the rest frame of the charge Q is consistent with the force in the rest frame of the wire. (2 points)