Statistical Mechanics

August 22, 2020

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

Consider a crystalline solid consisting of N non-interacting atoms. Assume that the atoms are located at fixed locations in the crystal lattice and the nuclei of the atoms have spin equal to one. Each nucleus has three allowed independent spin states labeled by the quantum number m, where $m = +1, 0, \text{or} -1$.

The energies of the states are zero in the $m = 0$ state and $\mu_N = \epsilon$ for both the $m = +1$ and $m = -1$ states, respectively, where μ_N is the nuclear magneton and ϵ is the field strength.

a.) Calculate the partition function for a single atom, Z_1, and for the whole system of N non-interacting atoms, Z.

b.) Determine the Helmholtz free energy.

c.) Calculate the total energy for the crystal. Simplify your answer to obtain the most compact expression.

Problem 2

The isothermal compressibility and entropy of a weakly interacting Bose gas at low temperatures can be approximated as

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T = \frac{V^2}{2a} \quad , \quad S(V,T) = \frac{5}{2}\, bVT^{3/2} \, , \tag{1}$$

where a and b are constants. In the limit of large volume and low temperature the internal energy density and pressure of the gas approach zero.

a.) Show that this gas obeys the third law of thermodynamics.

b.) Find the equation of state of the gas, $P(V,T)$.

c.) Calculate the internal energy of the gas, $E(V,T)$.

Problem 3

Consider a quantum system whose Hamiltonian changes sufficiently slowly such that the (quantum mechanical) adiabatic approximation is applicable. Prove that the entropy of an isolated statistical system governed by this Hamiltonian does not change.