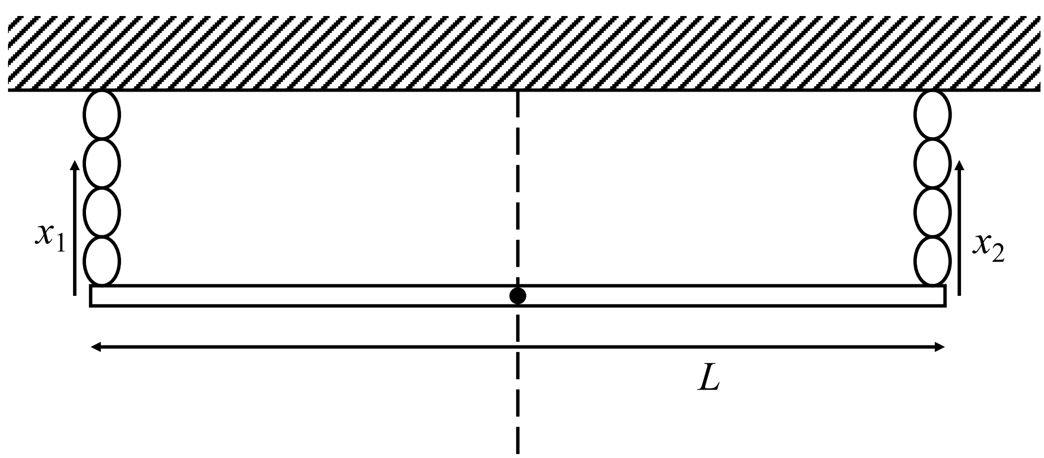
Classical Mechanics

August 21, 2021

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

A homogeneous rigid rod of mass M and length L is suspended by two identical massless springs, each with spring constant k. The system is subject to gravity. In equilibrium, the rod is stationary and horizontal, and the springs are vertical; their points of attachment to the ceiling are fixed. The center of mass of the rod (indicated by the black dot) is constrained to move vertically (as indicated by the dashed line); it cannot move left or right. Otherwise, the rod is free to move in the vertical plane (there is no motion in or out of the plane of the page).



The position of the rod is specified by the vertical distances x_1 and x_2 of the ends measured from the equilibrium position.

At $t = 0$, we have $x_1 = 0$, $\dot{x}_1 = 0$, $x_2 = x_0$, and $\dot{x}_2 = v_0$. In the limit of small oscillations, calculate the subsequent motion $x_1(t)$ and $x_2(t)$.

Problem 2

Consider a simple pendulum – a point mass m suspended by a string of negligible mass of length l. **Use the Lagrangian formulation** to find the force of constraint (the tension in the string) either as a function of time or position. Use the small angle approximation. (*Hint: You can use a force balance equation to check your result – but your main answer must be a derivation using the Lagrangian formulation.)*

Problem 3

Consider a straight tunnel connecting two cities that are 5000 km apart, as measured on the surface of the Earth, both at sea level. A rail is laid in the tunnel and a train car is allowed to roll, under only the force of gravity, starting at rest at one of the cities. The acceleration of gravity is $g = 9.8$ m/s^2 and the Earth's radius $R = 6400$ km. For the purposes of this exercise, the density of the Earth can be taken as constant.

a.) How long does it take for the car to arrive at the second city? Ignore friction, air resistance, and the rotation of the Earth. What will the velocity v of the car be when it reaches its destination?

b.) Now consider an additional, retarding force, some combination of friction and drag, that is proportional to v^2. Still ignoring the Earth's rotation, derive the equation of the phase-space trajectory and draw a rough sketch of what this looks like.