Statistical Mechanics

August 21, 2021

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

The bond distances in NH_3 are equal to 101.4 pm, and the bond angles are equal to 107.3°.

a.) Find the location of the center of mass and the principal moments of inertia.

b.) What is the symmetry number?

c.) Calculate the rotational partition function of NH_3 at 500 K.

Problem 2

At low densities, the virial expansion of the equation of state of a monatomic gas can be truncated at first order,

$$PV = NkT\left(1 + \frac{N}{V}B(T) + \dots\right)$$

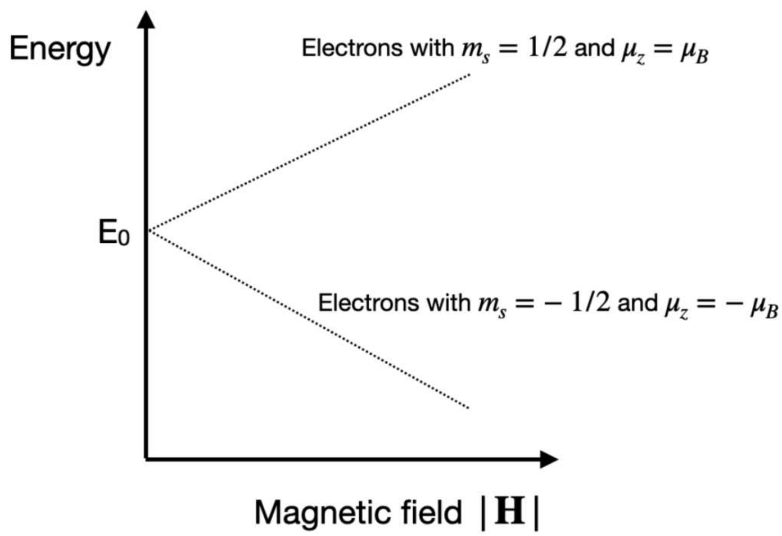
with a function of temperature $B(T)$ that you can assume to be given.

Also the heat capacity will have corrections to its ideal gas value. We can write it in the form

$$C_V = Nk\left(\frac{3}{2} - \frac{N}{V}D(T) + \dots\right)$$

Find the form of $D(T)$ in terms of $B(T)$ to make these two equations thermodynamically consistent, and give also the expressions for the entropy and the internal energy to this order in terms of $B(T)$.

Problem 3



Consider some number N of electrons with spin quantum number m_s either $+1/2$ or $-1/2$ (written as $\uparrow$ or $\downarrow$), split into two different energy levels via a uniform external magnetic field $\mathbf{H}$ as shown in the above figure. This energy splitting is caused by the alignment of the electrons in either a parallel or an antiparallel configuration with respect to the magnetic field direction. Assume $E_0 = 0$.

a.) Determine both $N_\downarrow$ and $N_\uparrow$, as well as the ratio $N_\downarrow/N_\uparrow$ using the expected Boltzmann distributions at a temperature T with a magnetic field of strength H.

b.) Determine the total magnetization M of the system as a function of N, H, and T. The magnetization is defined as $M = (N_\downarrow - N_\uparrow)\mu_B$. Eliminate any occurrence of $N_\downarrow$ or $N_\uparrow$ in your final equation.

c.) In the limit of $H \ll T$, simplify your equation. In this limit, the equation can be expressed as $M = C\frac{H}{T}$, where C is called the *Curie constant*. Determine an equation for the Curie constant C.

Hint: The Boltzmann distribution gives $n_i = Ae^{-\epsilon_i/kT}$, *where* A *satisfies* $\sum n_i = N$