Statistical Mechanics

August 20, 2022

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

Our universe is filled with black body radiation (photons) at a temperature of $T \sim 3\,\mathrm{K}$. This is thought to be a relic of early developments of the "Big Bang".

a.) Express the photon number density, n, analytically in terms of T and universal constants. Your answer must show explicitly the dependence on T and the universal constants. This expression may contain a dimensionless integral.

b.) Estimate the integral roughly and use your knowledge of the universal constants to estimate the photon density, n.

Problem 2

Consider a system of N identical but distinguishable particles. Each particle has a ground state with zero energy and an excited state with energy $\varepsilon > 0$.

a.) Find the partition function $Q(N, T)$ of this system.

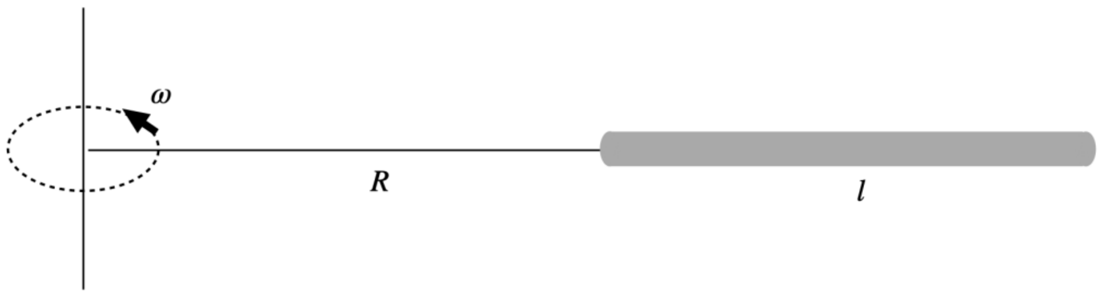
b.) Calculate the total internal energy $U(N, T)$ of this system.

c.) Estimate the value of $U(N, T)$ in the limits of $T \to 0$ and $T \to \infty$.

d.) Find the *minimum* and *maximum* values of U for this system (not necessarily in thermal equilibrium) and determine the range of energies corresponding to *positive* and *negative* temperatures.

Problem 3

Consider a classical ideal gas inside a closed long thin cylinder connected to a centrifuge as shown in the diagram below. The cylinder is separated from the rotation axis by a distance R and has a length l and a total volume V. The centrifuge then operates at an angular velocity ω.



a.) Take the (effective, centrifugal) potential energy on any molecule of gas as $u = -\frac{1}{2}m\omega^2 r^2$, where m is the mass of the particle and r is the distance from the rotation axis (disregard any Coriolis effects). Determine a partition function for N molecules of ideal gas in this system. Please evaluate any summation or integral in your final answer. *Note: For simplicity, it is admissible to approximate the tube as occupying a constant azimuthal angle in the plane of rotation, independent of distance from the axis.*

b.) From your answer in part a.), determine the total (effective) energy in the system.