

Classical Mechanics

Do two of the following three problems, each on a separate page (or pages) and write your name on every page you turn in.

Problem 1

In the presence of an isotropic central force $\vec{F} = f(r)\vec{e}_r$, a particle of mass m travels along the orbit

$$r(\varphi) = \frac{c}{\sqrt{\varphi}}$$

where r, φ denote polar coordinates in the plane of motion of the particle. Determine $\varphi(t)$ given the initial condition that, at $t = 0$, the particle is at $\varphi = 1$ traveling with angular velocity $\dot{\varphi} = 1$. What is the particle's speed at $t = 1$?

Problem 2

Two thin circular hoops of radius R , each with total mass m distributed homogeneously along the hoop, are placed in a single plane and glued together such that they touch only at a single point. Describe where the resulting object has its center of mass, and, invoking symmetry arguments, describe the principal axes for rotations around the center of mass. Give the corresponding principal moments. Consider rotation around an axis in the plane containing the hoops which goes through the center of mass and forms a 45° angle with the line connecting the centers of the hoops. If the angular speed of the rotation is ω , give the associated angular momentum vector and rotational kinetic energy.

Problem 3

This problem is given on a separate set of sheets (3 pages). Write your answers on the space provided on the sheets.