Classical Mechanics

Do two of the following three problems, each on a separate page (or pages) and write your name on every page you turn in.

Problem 1

A particle of mass m moving in the Kepler potential $V = -k/r$ achieves its closest approach to the force center, $r = r_0$, at $\varphi = 0$, where r, φ denote polar coordinates in the plane of motion of the particle. At $\varphi = \pi/3$, its distance from the force center is $r = 5r_0/4$. Determine the eccentricity ϵ of the orbit, the angular momentum, the energy, and the ratio of speeds $v(\varphi = \pi/3)/v(\varphi = 0)$.
Hint: If you're not completely confident in your knowledge of the bound Kepler orbits, the following factual detail may be helpful: The distance from the force center at $\varphi = \pi/2$ is determined purely by the angular momentum L via $mkr(\varphi = \pi/2) = L^2$.

Problem 2

A van of mass 1000 kg compresses its four springs by 10 cm when on level ground.

a) Calculate the spring constant for an individual spring.

b) Calculate the undamped oscillation period.

c) Shock absorbers are fitted to provide damping. Calculate the damping factor needed to obtain critical damping.

Problem 3

a) Find the velocity and position as a function of time for a particle of mass m which starts from rest at $x = 0$ assuming an applied force of form $F(t) = ct^2$.

b) Find the velocity as a function of distance for a particle of mass m which starts from rest at $x = 0$ assuming an applied force of $F(x) = F_0 + cx^2$.

c) Find the position as a function of velocity for a particle of mass m which starts at $x = 0$ and $v = v_0$ and is subject to a force $F(v) = c/v$.