Classical Mechanics

January 21, 2017

Work 2 (and only 2) of the 3 problems. Please put each problem solution on a separate sheet of paper and your name on each sheet.

Problem 1

A mass on a damped spring has (in a given system of units) a mass of 2, a damping force of 8 times the instantaneous speed, and a restoring force of 8 times the position. What type of damped spring motion is this? Be as specific as you can in characterizing the motion – for example, you might address its relationship to the equilibrium position. Also write down an equation that represents this motion.

Problem 2

Two people are holding the ends of a uniform plank of length l and mass m. One person suddenly lets go, while the other continues to hold the other end fixed. What is the initial downward acceleration of the free end? What is the upward force exerted by the second person just before and just after the release?

Problem 3

Consider a frictionless ball of mass m rotating in a vertical circular cone as shown in the following figure. The height of the cone z is proportional to the radius r with proportionality constant k. Assume the gravitational constant is g.

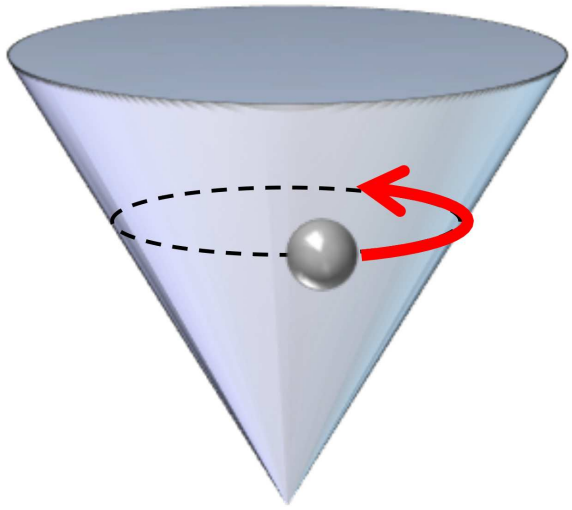

Figure 1: A ball rotating in a frictionless cone.

a.) Write the Lagrangian $\mathcal{L}$ in terms of variables r and the rotation angle ϕ.

b.) Write the equations of motion from the Lagrangian. Do not solve them.

c.) Prove that $mr^2\dot{\phi}$ is a constant of motion.

d.) What conserved quantity does this constant of motion refer to?

e.) Consider a harmonic *ansatz* such that $r(t) = Ae^{iat}$ and $\phi(t) = Be^{ibt}$. Apply this *ansatz* to just the constant of motion. What is the relationship between the frequencies a and b?