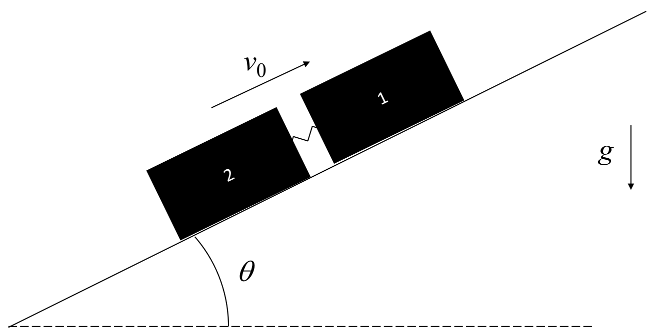
Classical Mechanics

Do <u>*two*</u> *of the following three problems, each on a separate sheet (or sheets). Attach each set to a provided cover sheet with your name, subject, and problem number.*

Problem 1

Two identical carts, each of mass m, are connected by a compressed spring. The spring stores potential energy E_s. At $t = 0$, the system of two carts is traveling up a hill, with incline angle θ, at an initial speed v_0; in this moment the spring is released, delivering all of its stored energy to the carts in the form of kinetic energy. The carts move without friction.

Assuming the hill extends indefinitely in both directions, compute the distance $d(t)$ between the two carts as a function of time. (You can take the initial distance between the carts to be zero.)



Problem 2

An object of mass m moves in an attractive potential $V = -c/\sqrt{r}$, where r denotes the distance from the origin and c is a constant. An experimenter is only able to take a few incomplete measurements of the object's motion, namely the x-component of its velocity at two different locations on its trajectory: At $\vec{r} = \vec{e}_x$, the experimenter finds $v_x = 5v_0$, and at $\vec{r} = \vec{e}_y$, the experimenter finds $v_x = 2v_0$, where $v_0 = \sqrt{c/(15m)}$. Will the object eventually escape from the potential or is the motion bound?

Problem 3

The magnitude of the Earth's gravitational force on a point mass is $F(r)$, where r is the distance from the Earth's center to the point mass. Assume the Earth is a homogenous sphere of radius R.

Suppose there is a very small shaft (a very thin well) in the Earth such that the point mass can be placed at a radius $r = R/2$. Calculate $\frac{F(R)}{F(R/2)}$. (Derive the formula for $F(r)$; don't just use a relationship you memorized.)