Classical Mechanics

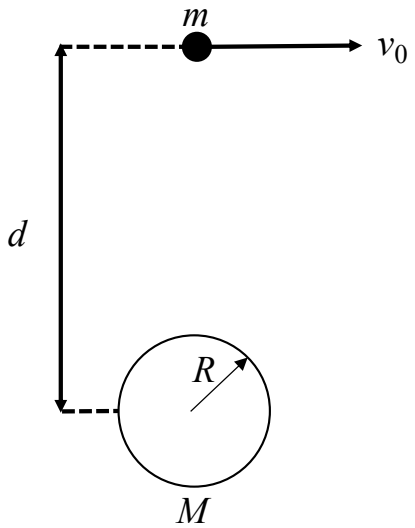
Do two of the following three problems, each on a separate sheet (or sheets). Attach each set to a provided cover sheet with your name, subject, and problem number.

Problem 1

Imagine we drill a hole through the Earth through the center to the other side. Derive an expression for the gravitational force as a function of distance from Earth's center, assuming uniform density. An object is dropped down the hole. Prove that the resultant motion is simple harmonic. Derive equations for its displacement, velocity, and acceleration as a function of time, and determine its period of motion.
Notes: Earth's radius is 6371 km, and mass is 5.98×10^{24} kg.

Problem 2

A satellite of mass m is in motion near a planet of mass M. At $t = 0$, the initial velocity of the planet is zero, and the initial velocity of the satellite has the value v_0 and is directed perpendicular to the direction from satellite to planet. The initial distance from satellite to planet is d. The planet has a radius R; the satellite is very small and we can ignore its size.



For what range of initial velocities $v_{\min} < v_0 < v_{\max}$ will the satellite be in a stable orbit about the planet? ("Stable orbit" means the satellite does not crash into the planet and also cannot run away to infinity.)

Problem 3

A particle of mass m starts with velocity v_0 from $x = 0$ and is subject to a force $F(v) \propto v^3$.

a) Find the position as a function of velocity

b) Find time as a function of position (hint: no need to solve for position as a function of time).