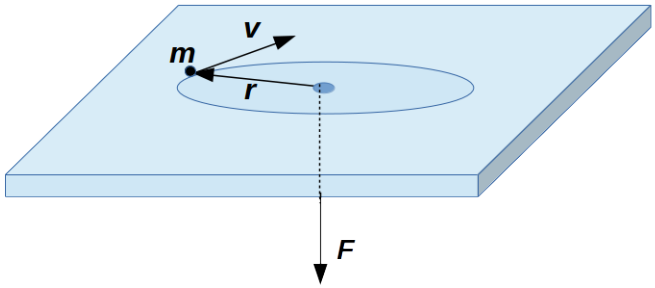
Classical Mechanics

Do two of the following three problems, each on a separate sheet (or sheets). Staple together the sheets for each problem, if using multiple sheets, but do not staple all problems together. Write at the top of the first sheet of each problem your name, subject, and problem number.

Problem 1

A string with a mass m attached to one end passes through a hole in a frictionless table . The mass rotates on the table with velocity v, kept in orbit, at a radius r, by the string. The string is then slowly pulled down from under the table with force F until the radius becomes $r/2$. Calculate the work done, in terms of the initial kinetic energy E_0 of the mass.



Problem 2

(a) A particle of mass m is suspended from a massless spring. The elastic constant of the spring is k. Show that the frequency of vertical oscillation of the mass is $\omega = \sqrt{k/m}$.

(b) Now suppose the mass of the spring is also m, the same as the mass of the particle. What is the frequency of vertical oscilation now? Assume the spring is of uniform mass density, and that the mass density remains uniform as the spring is stretched or compressed.

Problem 3

Consider a two-dimensional classical harmonic oscillator, where the potential energy is defined as $U(x, y) = \frac{1}{2}kr^2$, where $r = \sqrt{x^2 + y^2}$.

(a) From the potential energy, what is the magnitude of the acceleration of a particle of mass M at some position r?

The cartesian position for a two-dimensional harmonic oscillator at some time t can be expressed as $x = A\cos\left(\sqrt{\frac{k}{M}}t\right)$ and $y = B\cos\left(\sqrt{\frac{k}{M}}t - \phi\right)$, where ϕ is a phase difference between the x and y position.

In the case where the ***amplitudes are identical*** ($A = B$), evaluate the following:

(b) When $\phi = \pi/2$, what is the maximum, minimum, and average magnitude of the acceleration of the particle over a single period?

(c) When $\phi = 0$, what is the maximum, minimum, and average magnitude of the acceleration of the particle over a single period?

(d) In the case $\phi = 0$, a two-dimensional harmonic oscillator can be transformed into a one-dimensional harmonic oscillator with the solution $x' = A'\cos\left(\sqrt{\frac{k}{M}}t\right)$. Write what A' should be in terms of A, k, and/or M under this transformation.

Hints: $F = -\frac{dU(r)}{dr}$, $\cos(a - \pi/2) = \sin a$. For part **(c)**, the average acceleration over the time needed to go from the minimum $|a|$ to the first maximum $|a|$, is the same as the average acceleration over a single period.