Classical Mechanics

Do two of the following three problems, each on a separate sheet (or sheets). Staple together the sheets for each problem, if using multiple sheets, but do not staple all problems together. Write at the top of the first sheet of each problem your name, subject, and problem number.

Problem 1

A automobile of weight W moves up an incline at angle θ with power P. If the resistance per unit weight is κv, where v is the velocity and κ is a constant, find the maximum speed at which the automobile can go.

Problem 2

A system of two *coupled harmonic oscillators* is shown in Fig. 1. Two masses are connected to each other and to fixed rigid walls by springs. The masses are $3m$ and $2m$, with spring constants $6k$, $6k$, and $12k$ as shown. The masses are free to oscillate along the one-dimensional axis of the system.

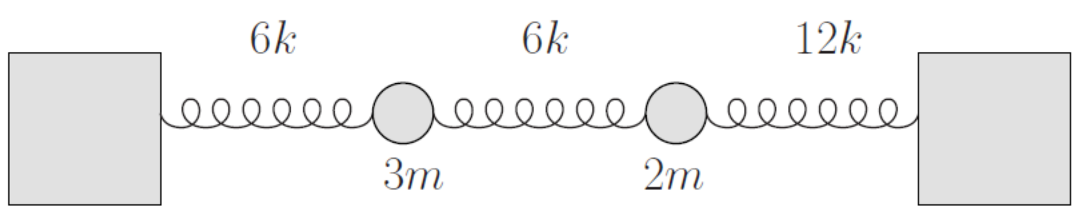

Figure 1: A system of two coupled harmonic oscillators.

(a) Suppose for a moment that $k = 1$ N/m and $m = 1$ kg. At some time t, the mass $3m$ is passing through its equilibrium position (zero displacement) with a velocity of 2 meters per second to the left, while the mass $2m$ is displaced 1 meter to the right and is at rest. What is the total energy of the system?

(b) From here on we will take k and m to be arbitrary constants. If the displacements of the masses from their equilibrium positions are written $x_1(t)$ and $x_2(t)$, write down the Lagrangian describing the system.

(c) Using either Newtonian or Lagrangian methods, find the **equations of motion** for the two masses.

Problem 3

Consider a vertical massless spring with spring constant k that at equilibrium extends a distance y from the ground. A plate with mass M is placed on the spring and it comes to a new equilibrium position of y_P. A massive rope with linear density ρ and length L is then held vertically from the top such that the bottom of the rope is barely touching the plate.

1. Write an expression for the displacement of the spring as a function of time after the rope is released until all the rope is collected on the plate. Hint: It may be useful to start by finding the displacement as a function of the distance between the top of the rope and the location it was released from.

2. Draw a graph that describes how the displacement of the spring changes over time, including the time *after* all of the rope has completely collected on the plate, up until the system reaches equilibrium again. This graph does not have to be exactly to scale, but should adequately show the trend of the motion.