Classical Mechanics

Do <u>two</u> of the following three problems, each on a separate sheet (or sheets). Staple together the sheets for each problem, if using multiple sheets, but do not staple all problems together. Write at the top of the first sheet of each problem your name, subject, and problem number.

Problem 1

Let's consider the difference in the gravitational force that the Moon exerts on a mass on the surface of the Earth and the force exerted on that same mass at the center of the earth, i.e. $\vec{F}_{\text{diff}} = \vec{F}(\vec{D} + \vec{R}) - \vec{F}(\vec{D})$. We can determine the maximum difference in force for the x-direction and the y-direction separately by looking at two scenarios:

1. For the x-direction: Show that the difference in the gravitational force of the Moon on an arbitrary mass m at point **a** and the same mass at point **c** is $F_{\text{diff-x}} = 2GMm\frac{R}{D^3}$. Hint: you can use the expansion $(x+1)^{-2} = 1 - 2x + 3x^2 - ...$ and neglect terms of $\left(\frac{R}{D}\right)^2$ or higher (since $D > R$ by about a factor of 50).
2. For the y-direction: Show that the total y-component of the gravitational force of the Moon on an arbitrary mass at point **b** is $F_{\text{diff-y}} = -GMm\frac{R}{D^3}$. Again neglect terms of $\left(\frac{R}{D}\right)^2$ or higher.
3. We can use this information to estimate the maximum height of the ocean tides. Calculate the work needed to bring an arbitrary mass from point **b** to point **c** (in the y-direction) and then from point **c** to point **a** (in the x-direction) using the F_{diff} given in parts 1 and 2. Next, using the work-energy theorem, let $W = mgh$ and calculate a numerical value for h.

Some useful constants are: $G = 6.67 \times 10^{-11}\ m^3/kg \cdot s^2$, $M_{Moon} = 7.35 \times 10^{22}\,kg$, $R_{Earth} = 6.37 \times 10^6\,m^2$, $g = 9.80\,m/s^2$, and $D = 3.84 \times 10^8\,m$.

Problem 2

A pendulum of length l with mass m on its end is moving through oil. The resistive force is $F_{\text{res}} = 2m\sqrt{g/l}(l\dot{\theta})$. The mass is pulled back to angle α and let go of with zero initial velocity. What is the resulting displacement as a function of time?

Problem 3

A chain of mass M and length L is hanging from one end over a scale, with its other end just touching the scale. Then the top end of the chain is released and the chain falls on the scale. Ignoring the length of the individual links of the chain, so that it can be treated as a continuous mass, derive an expression for the reading on the scale as a function of x, the length of the chain that has already fallen on the scale.