Classical Mechanics

Do two of the following three problems, each on a separate sheet (or sheets). Staple together the sheets for each problem, if using multiple sheets, but do not staple all problems together. Write at the top of the first sheet of each problem your name, subject, and problem number.

Problem 1

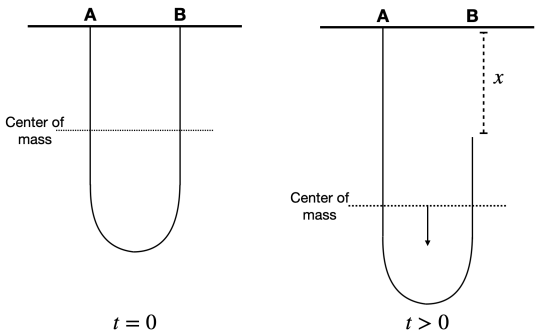

Figure 1: Position of the chain at $t = 0$ (left) and $t > 0$ (right)

A chain of constant linear density λ and total length l is attached at both ends and hung vertically as shown in the figure above. The chain is allowed to detach at point B, and that end is allowed to fall vertically.

1. (2 pt) What is the tension at point A at $t = 0$ and what is the intial potential energy of the system?

2. (2 pt) Determine the potential energy in the system as the chain-end falls and separates from point B at $t > 0$ as a function of x.

3. (3 pt) Find the velocity and acceleration of the center-of-mass of the falling section of chain as a function of x.

4. (3 pt) Find the tension in the chain at point A as a function of x while the chain end is falling ($t > 0$). Comment on the tension when $x \approx l$.

Problem 2

Consider a thin rod with $I = ma^2/3$. An impulse, P, is applied a distance b from the center of the rod. Find the total energy in terms of the impulse, P, and the point of impact, b.

Problem 3

A bead of mass m on a frictionless table is attached to the end of a massless string, as in the figure below. The string passes through a hole in the middle of the table and is pulled downwards so that it is always taut. The mass initially moves in a circle of radius R, with kinetic energy E. As the string is slowly pulled, the radius decreases until it is equal to $R/2$. Calculate how much work was done.

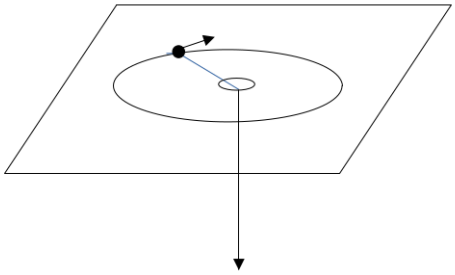

Figure 2: Bead tied to a string on a table