Electricity and Magnetism

Do two of the following three problems, each on a separate page (or pages) and write your name on every page you turn in.

Problem 1

An infinitely long, straight, cylindrical conducting wire of radius a has a cylindrical hole of radius b passing through its body. The axis of the hole is located at a distance d from the axis of the wire ($d + b < a$). A steady current $\mathbf{I}$ flows through the conducting wire. The current is uniformly distributed over the cross section of the wire. Find the magnitude of the magnetic field inside the hole.

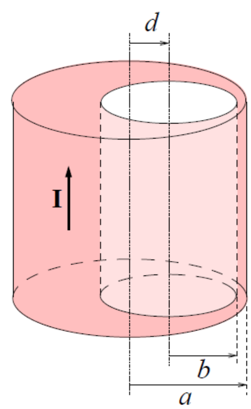

Figure 1: Problem 1 configuration

Problem 2

a. Consider an infinitely long solid cylinder of radius R carrying a uniform charge density ρ_0. Find the electric field inside the cylinder using the integral form of Gauß' law and show that the electric potential inside the cylinder can be written as

$$V(r) = -\frac{\rho_0 r^2}{4\epsilon_0}$$

(with r denoting the distance from the cylinder axis).

b. Now let the cylinder carry a uniform current density $\vec{J}$ directed along the cylinder. Neglecting the possibility of magnetization of the cylinder material, find the magnetic field inside the cylinder using the integral form of Ampère's law.

c. Continuing with the same setup, determine the vector field $\vec{A}$ inside the cylinder (*Hint*: the result shown in question (a) may be useful for this), and derive the magnetic field inside the cylinder from $\vec{A}$.

Problem 3

Two small spheres, each carrying a charge q, are attached to the ends of a light rod of length d, which is suspended from the ceiling by a thin torsion-free fiber (see Figure 2). There is a uniform magnetic field **B** pointing straight down, in the cylindrical region of radius R. The system is initially at rest. Compute the angular momentum, when the magnetic field is switched off.

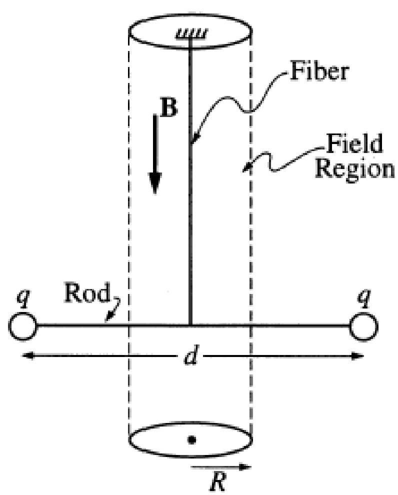

Figure 2: Arrangement for Problem 3