Electricity and Magnetism

Do two of the following three problems, each on a separate sheet (or sheets). Attach each set to a provided cover sheet with your name, subject, and problem number.

Problem 1

Unpolarized light is traveling along the $+\hat{z}$ axis. In the following questions, assume ideal, perfectly polarizing filters.

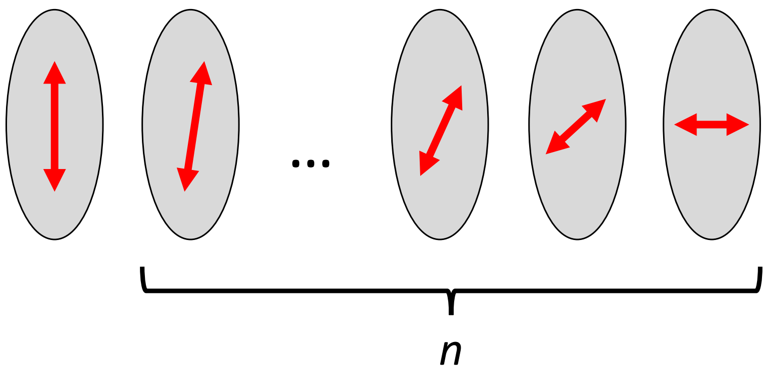

Figure 1: Polarizers

1. A first (0^{th}) polarizer has its polarization axis aligned along $\hat{x}$. What is the intensity of transmitted light? What is the size and direction of the electric field relative to the initial light? Show your derivation.

2. Now, a pair of polarizers is placed immediately after the 0^{th} polarizer. The first is oriented 45° relative to the first i.e., $(\hat{x} + \hat{y})/\sqrt{2}$, and the second is at 90° relative to the first i.e., $\hat{y}$. What is the intensity of the transmitted light?

3. Now suppose n polarizers are placed after the 0^{th} polarizer. The j^{th} polarizer is rotated by $j\frac{\pi}{2n}$ relative to the $\hat{x}$ axis ($j = 1$ to n). See Fig. 1. What is the intensity of the transmitted light? Make an argument for why the light intensity increases as n increases.

4. As $n \to \infty$, what is the transmitted intensity? Show your derivation and discuss the significance of the answer.

Problem 2

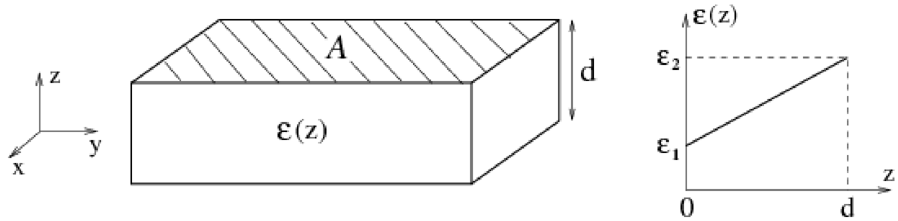

Figure 2: Capacitors

A capacitor is constructed from two parallel rectangular metal plates of area A separated by distance d. The space between the plates is filled with a dielectric material. The dielectric permittivity of the material inside the capacitor linearly changes from ϵ_1 to ϵ_2. Calculate the capacitance of this parallel plate capacitor neglecting edge effects.

Problem 3

A small electrically charged bead with the mass m and charge Q can slide on a circular insulating string without friction. The radius of the circle is r. A point-like electric dipole is at the center of the circle with the dipole moment P lying in the plane of the circle. Initially the bead is at an angle $\theta = \frac{\pi}{2} + \delta$, where δ is infinitely small, as shown schematically in the figure below.

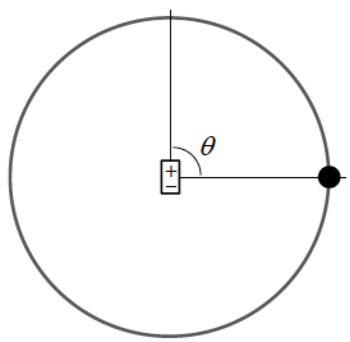

Figure 3: Bead on a string

(a) How does the bead move after it is released? Find the bead velocity as a function of the angle θ.

(b) Find the normal force exerted by the string on the bead.