Electricity and Magnetism

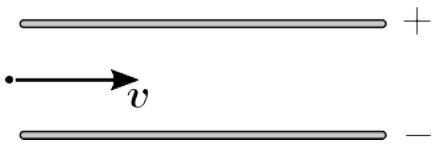
Do two of the following three problems, each on a separate sheet (or sheets). Attach each set to a provided cover sheet with your name, subject, and problem number.

Problem 1

A molecule consists of two atoms a distance, a, from each other. The atoms are spherically symmetric, and their polarizabilities are β' and β''. Find the polarizability tensor for the molecule assuming that the atomic radii are small compared with a. In particular, discuss the case where $\beta' = \beta''$.

Problem 2

A fast (but nonrelativistic) electron is passing through the gap between two oppositely charged parallel plates in the direction parallel to the plates. The potential between the plates is V, the distance between the plates is d and their length along the direction of the electron velocity is l. The charge of the electron is q_e and the mass is m_e.



(a) Find the condition on the velocity of the electron (v much larger or smaller than what?) so that the trajectory of the electron can be still considered approximately as a straight line.

(b) Find the total amount of electromagnetic radiation emitted by the electron under this condition.

(c) Draw the directional dependence of this radiation. What is the direction (or directions) in which the radiation is minimal?

Problem 3

You have an ohmic material with varying conductance as a function of position. You build a cylinder of radius a and length L (starting from $z = 0$ to $z = L$) and send current along its length. The ends are at constant potentials (0 and V).

1. If the conductance is $\sigma(z) = k(z + 1)^m$ (i.e., only a function of z), then what is the resistance of the cylinder?

2. If the conductance is $\sigma(s) = ks^m$ (i.e., only a function of the cylindrical radius), then what is the resistance of the cylinder?

3. In the general problem where $\sigma(\vec{r})$ is arbitrary, you calculate the current crossing a right circular cross section at z to be $I \propto z^n$ where n is a constant. What is the value of n? Why?

4. Prove that the charge density ρ may not be zero in steady state for a non-constant conductance σ.