Electricity and Magnetism

Do <u>*two*</u> *of the following three problems, each on a separate sheet (or sheets). Staple together the sheets for each problem, if using multiple sheets, but do not staple all problems together. Write at the top of the first sheet of each problem your name, subject, and problem number.*

Problem 1

A charged parallel-plate capacitor with a uniform electric field $\overrightarrow{E} = E\hat{z}$ is placed in a uniform magnetic field $\overrightarrow{B} = B\hat{y}$ pointing parallel to the capacitor plates (the capacitor plates are parallel to the xy plane). The spacing between the plates is d and the area of each plate is A.

(a) What is the Poynting vector between the plates?

(b) Find the electromagnetic field momentum between the plates.

(c) Consider now a resistive wire connecting the two plates along the z axis, so the capacitor slowly discharges. The current through the wire will experience a magnetic force; what is the total impulse (change in momentum) delivered to the system during the discharge?

Problem 2

Consider a static electric field given by

$$\vec{E}(x,y,z) = E_0\Big(4xy^2\,\hat{\imath} + 4y(x^2 - y^2)\,\hat{\jmath} - 18z^2\,\hat{k}\Big) \tag{1}$$

where E_0 is a constant. The Cartesian coordinates (x, y, z) may be considered unitless.

(a) Use the differential form of Gauss' law to find the **charge density** $\rho(x, y, z)$ which produces the electric field given in Eq. 1

(b) Find an **electrostatic scalar potential** $\phi(x, y, z)$ corresponding to the electric field $\vec{E}$. You may choose the arbitrary zero value of the potential however you like.

Now, consider a static magnetic field given by

$$\vec{B}(x,y,z) = B_0\Big((-2z + xz)\,\hat{\imath} - yz\,\hat{\jmath} - 2x\hat{k}\Big) \tag{2}$$

where B_0 is a constant. The Cartesian coordinates (x, y, z) may be considered unitless.

(c) Use Ampère' law to find the **current density** $\vec{J}(x, y, z)$ which produces the magnetic field given in Eq. 2.

Problem 3

Determine the electric field produced at any point in space by an infinite line of charge with a uniform linear charge density λ.