Electricity and Magnetism

Do two of the following three problems, each on a separate sheet (or sheets). Staple together the sheets for each problem, if using multiple sheets, but do not staple all problems together. Write at the top of the first sheet of each problem your name, subject, and problem number.

Problem 1

a.) Consider an infinitely long solid cylinder of radius R carrying a uniform charge density ρ_0. Find the electric field inside the cylinder using the integral form of Gauß' law and show that the electric potential inside the cylinder can be written as

$$V(r) = -\frac{\rho_0 r^2}{4\epsilon_0}$$

(with r denoting the distance from the cylinder axis).

b.) Let the same cylinder now instead be electrically neutral, but carry a uniform current density $\vec{J}$ directed along the cylinder. The cylinder material cannot be magnetized. Give the vector potential $\vec{A}$ inside the cylinder (*Hint*: you may be able to exploit an analogy with part a.)), and derive the magnetic field inside the cylinder from $\vec{A}$.

Problem 2

A wire is formed into the shape of a square of edge length L. Show that when the current in the loop is I, the magnetic field at point P, a distance x from the center of the square along its axis is given by

$$B = \frac{\mu_0 I L^2}{2\pi \left(x^2 + \frac{L^2}{4}\right) \sqrt{x^2 + \frac{L^2}{2}}}.$$

Problem 3

The average intensity of the solar radiation reaching the Earth, summed over the entire electromagnetic spectrum, is $I \simeq 1.36\times10^3$ W/m^2; this quantity is known as "the solar constant." Calculate the amplitudes of the electric and magnetic fields of a plane electromagnetic wave that carries this amount of power per unit area.