Electricity and Magnetism

Do two of the following three problems, each on a separate sheet (or sheets). Staple together the sheets for each problem, if using multiple sheets, but do not staple all problems together. Write at the top of the first sheet of each problem your name, subject, and problem number.

Problem 1

Consider the following electric field in cylindrical coordinates (ρ, ϕ, z), constant in time t:

$$\vec{E} = \frac{1}{1+\rho+z^2}\vec{e_\rho}$$

What charge and current densities are needed for this electric field to exist? You may assume that there are no currents at $t = 0$.

Problem 2

A homogeneous material of dielectric constant ε has a spherical cavity of radius R, which is in vacuum, with a point electric dipole $\vec{p}$ at its center. Find the solution for the electric field everywhere.

Problem 3

A long, conducting, uncharged, thin-walled cylindrical shell with radius a is oriented with its axis in the z-direction. Parallel to the cylinder, at a distance R from its axis $(R > a)$ is a long, thin wire that carries a uniform, linear charge density $\lambda > 0$. Use the method of images:

a) What conditions must be met by the surface charge? Find the potential on the surface of the cylinder if the potential at infinity is zero.

b) Find the potential in cylindrical coordinates ρ, φ at any point on the x-y plane outside the cylinder.