Electricity and Magnetism

Do two of the following three problems, each on a separate sheet (or sheets). Staple together the sheets for each problem, if using multiple sheets, but do not staple all problems together. Write at the top of the first sheet of each problem your name, subject, and problem number.

Problem 1

In the lab, you discover a nifty device called an electromagnetic meter. It measures either the electric or the magnetic field locally at the position of the device. There is a toggle switch allowing you to choose either electric or magnetic mode. Unfortunately, through heavy use, the labeling of the switch has worn off. Nonetheless, you take the meter as you found it and map out the field

$$\vec{F} = \frac{\rho}{1+\rho+z^2}\vec{e_\phi}$$

around the lab space, where (ρ, ϕ, z) denote cylindrical coordinates. Note that the lab space is not necessarily empty, you cannot exclude the possibility that there could be charges and currents where you measure; however, the situation is static — you don't have to worry about the fields changing during the time it takes you to move around the lab space to carry out your measurements. Was the field $\vec{F}$ you measured the electric field, or was it the magnetic field (or is it impossible to decide)? Justify your answer.

Problem 2

1. A semi-spherical surface (defined by $r = R$, $0 \leq \theta \leq \pi/2$, $0 \leq \varphi \leq 2\pi$) is evenly charged with Q.

(1) Please find out the electric potential V at the center.

(2) Please find out the electric field $\mathbf{E}$ at the center.

(3) Prove that, the electric field $\mathbf{E}$ on the disk (defined by $r \leq R$, $\theta = \pi/2$, $0 \leq \varphi \leq 2\pi$) is the same everywhere.

Problem 3

An electric charge distribution produces an electric field

$$\hat{E} = c\left(1 - e^{-\alpha r}\right)\frac{\hat{r}}{r^2},$$

where c and α are constants. Find the total charge within a radius $r = 1/\alpha$.