Modern Physics

Do <u>two</u> of the following three problems, each on a separate page (or pages) and write your name on every page you turn in.

Problem 1

Assume that a hydrogen atom at $t = 0$ is in a state described by the following wave function:

$$\psi(\vec{r}, 0) = \frac{1}{\sqrt{10}} \left(2\psi_{100} + \psi_{210} + \sqrt{2}\psi_{211} + \sqrt{3}\psi_{21-1} \right),$$

where the subscripts are values of the quantum numbers n, l, m. Ignore spin and relativistic effects. Some hints:

$$R_{10} = 2\gamma^{\frac{3}{2}} e^{-\frac{r}{a_0}}$$

and

$$R_{21} = \frac{2}{\sqrt{3}} \gamma^{\frac{5}{2}} r e^{-\frac{r}{2a_0}},$$

where $a_0 = 0.53$ Å is the Bohr radius and $\gamma = \frac{1}{a_0 n}$, where n is the principal quantum number;

$$\begin{aligned} L_x &= \frac{L_+ + L_-}{2}, \\ L_+ Y_{lm} &= \sqrt{(l+m+1)(l-m)} Y_{lm+1}, \\ L_- Y_{lm} &= \sqrt{(l-m+1)(l+m)} Y_{lm-1}. \end{aligned}$$

a) What is the expectation value for the energy of this system? Express your answer in eV.

b) What is the probability of finding this system in the state with $l = 1$ and $m = +1$ as a function of time?

c) What is the probability of finding the electron within one Bohr radius of the proton at $t = 0$? A good approximate result is sufficient.

d) Suppose that a measurement results in $l = 1$ and $l_x = +1$, where l_x is the eigenvalue of the L_x operator. Describe the wavefunction immediately after the measurement in terms of the ψ_{nlm} given above.

Problem 2

A certain system has a ground state (E_0) and first-excited state (E_1) that are very nearly degenerate; the difference in energy between them is very small.

$$E_1 - E_0 \ll E_0$$

All of the other states of the system (second-excited, third-excited, etc.) are much higher in energy and we will ignore them.

A small external field A is introduced, which produces an interaction between the ground and first-excited states:

$$\langle 1|A|0\rangle = 3Da$$

where a is a small positive constant with units of energy. ("Small" means $a \ll E_0$.)

What is the change in the energies of the ground and first-excited states, to first order in a?

Problem 3

A beam of particles of mass m moving along the x-axis with momentum p is incident from the left on a δ-function potential barrier at $x = 0$:

$$V(x) = b\delta(x),$$

where $b > 0$; the potential is zero everywhere else. Calculate the transmission probability of the beam through the barrier as a function of particle velocity, assumed non-relativistic.