Modern Physics

Do two of the following three problems, each on a separate sheet (or sheets). Attach each set to a provided cover sheet with your name, subject, and problem number.

Problem 1

Consider a one-dimensional, simple harmonic oscillator with an additional, anharmonic x^4 term:

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \lambda x^4.$$

a) What are the energy eigenvalues of H when $\lambda = 0$?

b) What is the shift in the lowest eigenvalue found in part **(a)** to first order in λ?

c) Find this energy shift in λ^2

d) Using a Gaussian trial function, make a variational estimate of the energy of the ground state. (You need only obtain an explicit equation whose solution would give this estimate.) Solve this equation to first order in λ and compare your result with that in part **(b)**

Problem 2

A particle of mass M is in a one-dimensional harmonic oscillator potential, $V = \frac{1}{2}kx^2$.

a) Show that the ground-state wavefunction is of Gaussian shape, $\psi(x) = A\exp(-Bx^2)$ and determine the parameters A and B

b) Assume that the system is initially in the ground state. The spring constant is suddenly doubled, $k \to 2k$ so that the *new* potential is $V = kx^2$. The particle's energy is then measured. What is the probability for finding the system in the ground state of the *new* potential?

Useful equation:

$$\int_0^\infty \exp\left(-a^2x^2\right) = \frac{\sqrt{\pi}}{2a}$$

Problem 3

a) A photon moving in the z direction is in a state $|\psi\rangle = \alpha\,|\mathcal{X}\rangle + \beta\,|\mathcal{Y}\rangle$, where $|\mathcal{X}\rangle$ and $|\mathcal{Y}\rangle$ are the states with linear polarization along the x and y direction, respectively.

1. The photon passes first through a linear polarizer with the polarization axis oriented in the y direction, then one rotated by $45°$ in the $x-y$ plane and finally a third one with the axis oriented in the x direction. Calculate, in terms of α and β, the probablity that the photon will emerge from the third polarizer.

2. What would the probability be if the middle polarizer were absent? Explain the difference.

b) Consider the case where the coefficients in **(a)** have the specific values $\alpha = \frac{1}{\sqrt{2}}$ and $\beta = \frac{i}{\sqrt{2}}$. Show that this photon is in an eigenstate of the J_z operator (z-component of the angular momentum). What is the eigenvalue?
Hint: Try doing a rotation of the state by some angle ϕ around z, first by rotating explicitly the $|\mathcal{X}\rangle$ and $|\mathcal{Y}\rangle$ components and then by using the rotation operator and compare the two results. Recall that J_z is the generator of rotations.