Modern Physics

Do <u>two</u> of the following three problems, each on a separate sheet (or sheets). Attach each set to a provided cover sheet with your name, subject, and problem number.

Problem 1

A centrally symmetric potential gives rise to a discrete set of energy eigenvalues. Show that the minimum energy for a given l (l being the orbital quantum number) increases with increasing l.
Hints:

- Consider the Ritz variational principle (variational method)
- The radial part of the 3D Schrödinger equation is

$$-\frac{\hbar^2}{2m_0 r}\frac{d^2}{dr^2}[rR(r)] + \frac{\hbar^2 l(l+1)}{2m_0 r^2}R(r) + V(r)R(r) = ER(r)$$

Problem 2

In a given representation, a particle is in the state

$$\psi = \frac{1}{2}\begin{pmatrix} \sqrt{3} \\ i \end{pmatrix} .$$

Suppose first a measurement of

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

followed by a measurement of

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

is performed, and the outcomes of the measurements are logged. List the possible laboratory logs which may result, and the respective probabilities with which these logs are obtained.

Problem 3

Consider a quantum mechanical particle of mass m in a one dimensional potential

$$V(x) = \frac{\hbar^2 v}{2ma} \left[-\delta(x) + \delta(x-a) \right] ,$$

with a and v positive constants.

- Solve the stationary Schrödinger equation for a negative energy $E = -\frac{\hbar^2\kappa^2}{2m}$, $\kappa > 0$, for suitable boundary conditions. Derive the following equation for κ : (7.5 points)

$$\mathrm{e}^{2\kappa a} = \frac{1}{1 - \left(\frac{2\kappa a}{v}\right)^2} .$$

- Show that there is one and only one bound state in this problem which is independent of v. Find an argument that the energy of that bound state is proportional to $-v^2$ for large values of v. (2.5 points)